

# Homework Assignment 6

Due: Wednesday, November 22

1. Prove that any two consecutive Fibonacci numbers are relatively prime.
2. Find the smallest positive integer  $a$  for which the equation

$$1602 \cdot x + 1170 \cdot y = 10^6 + a$$

has a solution in integers  $x$  and  $y$ . Also, find one such solution. Hint: what do you know about the set  $\{1602x + 1170y \mid x, y \in \mathbb{Z}\}$ ?

3. (a) For natural numbers  $x$  and  $y$ , must it be true that  $2^x \equiv 2^y \pmod{n}$  if  $x \equiv y \pmod{n}$ ?  
(b) Show that  $2^{70} + 3^{70}$  is divisible by 13.  
(c) Calculate the last two digits of  $3^{1200}$ .
4. Suppose that  $n > 1$  is a positive integer greater such that  $2^n + n^2$  is prime. Show that  $n$  is divisible by 3. (Hint: work in mod 6.)