Name: Solutions

Quiz 6

Math 54 - Summer 2008
July 18, 2008

1. If possible, diagonalize the matrix

$$A = \begin{bmatrix} 3 & 0 & -1 \\ -1 & -1 & 0 \\ -1 & 0 & 3 \end{bmatrix}.$$

To find eigenvalues:

$$\begin{aligned} 0 &= \det(A-\lambda I) \\ &= (-1-\lambda)\cdot\left[(3-\lambda)^2-1\right] \\ &= -(\lambda+1)(\lambda^2-6\lambda+8) \\ &= -(\lambda+1)(\lambda-4)(\lambda-2). \end{aligned}$$

Thus the eigenvalues of A are $-1, 2, 4$. To calculate eigenspaces:

$$E_{-1}(A) = \mathrm{Nul}\begin{bmatrix} 4 & 0 & -1 \\ -1 & 0 & 0 \\ -1 & 0 & 4 \end{bmatrix} = \mathrm{span}\left\{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right\},$$

$$E_{2}(A) = \mathrm{Nul}\begin{bmatrix} 1 & 0 & -1 \\ -1 & -3 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \mathrm{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right\}, \text{ and}$$

$$E_{4}(A) = \mathrm{Nul}\begin{bmatrix} -1 & 0 & -1 \\ -1 & -5 & 0 \\ -1 & 0 & -1 \end{bmatrix} = \mathrm{span}\left\{\begin{bmatrix} -5 \\ 1 \\ 5 \end{bmatrix}\right\}.$$

Thus $A = PDP^{-1}$, where

$$P = \begin{bmatrix} 0 & 1 & -5 \\ 1 & 0 & 1 \\ 0 & 1 & 5 \end{bmatrix}, \quad D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

2. Suppose that A is diagonalizable, and $A^8 + 2A^2 = 0$. What can you say about A? Prove your answer.

Suppose that P is invertible and D is diagonal such that $A = PDP^{-1}$. Let $D = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$. We have $0 = A^8 + 2A^2 = PD^8P^{-1} + 2PD^2P^{-1} = P\cdot[D^8+2D^2]P^{-1}$. Thus $D^8 + 2D^2 = P^{-1}\cdot 0 \cdot P = 0$. Since

$$D^8 + 2D^2 = \begin{bmatrix} \lambda_1^8+2\lambda_1^2 & & \\ & \ddots & \\ & & \lambda_n^8+2\lambda_n^2 \end{bmatrix},$$

we see that $\lambda_i^8 + 2\lambda_i^2 = 0$ for each i. Thus $\lambda_i = 0$ for each i. Thus $D = 0$. Thus $A = PDP^{-1} = 0$.