Name: Solutions

Quiz 5

Math 54 - Summer 2008

July 16, 2008

1. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 7 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \end{bmatrix} \right\}$$

Find the standard matrix of the linear map on $\mathbb{R}^2$ given by $\mathbf{x} \mapsto [\mathbf{x}]_{\mathcal{B}}$.

Notice that $[e_1]_{\mathcal{B}} = \frac{1}{29}\begin{bmatrix} 3 \\ -2 \end{bmatrix}$, since $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{3}{29}\begin{bmatrix} 7 \\ 2 \end{bmatrix} - \frac{2}{29}\begin{bmatrix} -4 \\ 3 \end{bmatrix}$,

and $[e_2]_{\mathcal{B}} = \frac{1}{29}\begin{bmatrix} 4 \\ 7 \end{bmatrix}$, since $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{4}{29}\begin{bmatrix} 7 \\ 2 \end{bmatrix} + \frac{7}{29}\begin{bmatrix} -4 \\ 3 \end{bmatrix}$.

Thus the standard matrix of the map $\underline{x} \mapsto [\underline{x}]_{\mathcal{B}}$

is $\frac{1}{29}\begin{bmatrix} 3 & 4 \\ -2 & 7 \end{bmatrix}$.

2. Let $\mathcal{C}$ be a basis for $\mathbb{R}^n$ and A be an n-by-n matrix. Show that if $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ satisfy the equation $A\mathbf{x} = \mathbf{y}$ then they also satisfy $B[\mathbf{x}]_{\mathcal{C}} = [\mathbf{y}]_{\mathcal{C}}$, where $B = P_{\mathcal{C}}^{-1} A P_{\mathcal{C}}$.

If $A\underline{x} = \underline{y}$, then
$$\begin{aligned} B[\underline{x}]_{\mathcal{C}} &= P_{\mathcal{C}}^{-1} A P_{\mathcal{C}} [\underline{x}]_{\mathcal{C}} \\ &= P_{\mathcal{C}}^{-1} A(\underline{x}) && (\text{since } \underline{x} = P_{\mathcal{C}}[\underline{x}]_{\mathcal{C}}) \\ &= P_{\mathcal{C}}^{-1} y && (\text{since } A\underline{x} = \underline{y}) \\ &= [y]_{\mathcal{C}}. && (\text{since } P_{\mathcal{C}}^{-1} y = [y]_{\mathcal{C}}) \end{aligned}$$