

Name:

Solution

Quiz 4

Math 54 - Summer 2008

July 8, 2008

1. Compute the determinant of the matrix

$$A = \begin{bmatrix} 2 & 3 & -2 & 1 & 1 \\ 0 & 0 & -2 & 1 & 6 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 1 & 0 & -1 & -2 \\ -2 & -6 & 4 & -1 & 3 \end{bmatrix}$$

Make certain that any reader could follow the calculation.

Perform $R_1 + R_5 \rightarrow R_5$ to get:

$$B = \begin{bmatrix} 2 & 3 & -2 & 1 & 1 \\ 0 & 0 & -2 & 1 & 6 \\ 0 & 0 & 1 & 3 & 4 \\ 0 & 1 & 0 & -1 & -2 \\ 0 & -3 & 2 & 0 & 4 \end{bmatrix}$$

($\det A = \det B$)

Expand down first column:

$\det B = 2 \cdot \det C$, where

$$C = \begin{bmatrix} 0 & -2 & 1 & 6 \\ 0 & 1 & 3 & 4 \\ 1 & 0 & -1 & -2 \\ -3 & 2 & 0 & 4 \end{bmatrix}$$

Perform $3R_3 + R_4 \rightarrow R_4$ to get:

$$D = \begin{bmatrix} 0 & -2 & 1 & 6 \\ 0 & 1 & 3 & 4 \\ 1 & 0 & -1 & -2 \\ 0 & 2 & -3 & -2 \end{bmatrix}$$

($\det C = \det D$)

Thus $\det D = \det E$, where

$$E = \begin{bmatrix} -2 & 1 & 6 \\ 1 & 3 & 4 \\ 2 & -3 & -2 \end{bmatrix}$$

Perform $2R_2 + R_1 \rightarrow R_1$
and $-2R_2 + R_3 \rightarrow R_3$:

$$F = \begin{bmatrix} 0 & 7 & 14 \\ 1 & 3 & 4 \\ 0 & -9 & -10 \end{bmatrix}, \quad (\det E = \det F)$$

Now

$$\det F = -\det \begin{bmatrix} 7 & 14 \\ -9 & -10 \end{bmatrix}$$

$$= -[(-70) - (-126)]$$

$$= -56$$

$$\text{Thus } \det A = 2 \cdot (-56) = -112.$$