Name: Solutions

Quiz 3

Math 54 - Summer 2008

July 8, 2008

1. Determine whether the matrix

$$A = \begin{bmatrix} 1 & 3 & 0 \\ -2 & -3 & 1 \\ 3 & -3 & 0 \end{bmatrix}$$

is invertible, and find its inverse if it has one.

Form the augmented matrix:

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ -2 & -3 & 1 & 0 & 1 & 0 \\ 3 & -3 & 0 & 0 & 0 & 1 \end{array}\right]$$

$2R_1+R_2 \to R_2$, $-3R_1+R_3 \to R_3$:

$$\left[\begin{array}{cccccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 2 & 1 & 0 \\ 0 & -12 & 0 & -3 & 0 & 1 \end{array}\right]$$

$-R_2+R_1 \to R_1$, $4R_2+R_3 \to R_3$:

$$\left[\begin{array}{cccccc} 1 & 0 & -1 & -1 & -1 & 0 \\ 0 & 3 & 1 & 2 & 1 & 0 \\ 0 & 0 & 4 & 5 & 4 & 1 \end{array}\right]$$

$\frac{1}{4}R_3+R_1 \to R_1$; $-\frac{1}{4}R_3+R_2 \to R_2$:

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 1/4 & 0 & 1/4 \\ 0 & 3 & 0 & 3/4 & 0 & -1/4 \\ 0 & 0 & 4 & 5 & 4 & 1 \end{array}\right]$$

$1/3 R_2 \to R_2$, $1/4 R_3 \to R_3$:

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 1/4 & 0 & 1/4 \\ 0 & 1 & 0 & 1/4 & 0 & -1/12 \\ 0 & 0 & 1 & 5/4 & 1 & 1/4 \end{array}\right]$$

Thus A^{-1} exists, and equals

$$\begin{bmatrix} 1/4 & 0 & 1/4 \\ 1/4 & 0 & -1/12 \\ 5/4 & 1 & 1/4 \end{bmatrix}.$$

2. Suppose that A is a square matrix such that $\text{rank}(A) = 1$ and that the first column of A has a nonzero entry. Prove that each column of A is a multiple of the first column of A.

Let $\underline{a}$ be the first column of A. Since $\underline{a} \neq 0$, the set $\{\underline{a}\}$ is linearly independent. Since $1 = \text{rank}(A) = \dim(\text{Col}(A))$, $\{\underline{a}\}$ is a basis for $\text{Col}(A)$. Thus every column of A belongs to $\text{span}\{\underline{a}\}$, which is the set of multiples of $\underline{a}$. ∎