Math 54 - Midterm (Sample) #1

July 11, 2008, 08:00-10:00

Name: ____

This is a closed book, closed notes exam. Calculators are not allowed. You have two hours to complete the exam. To receive full credit, write legibly, show your work and write proofs in complete sentences. If you need more space, use the back of the page of the problem on which you are working.

Problem	Points	Your Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
Total	120	

1. Let

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 3 & 3 & 0 & -2 \\ -2 & -1 & 5 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}.$$

(a) Find all solutions $\mathbf{x} \in \mathbb{R}^4$, if any, of the equation $A\mathbf{x} = \mathbf{b}$. (14 points)

(b) Is the linear map L_A onto? Explain. (6 points)

2. Assume that $A \in \mathbb{M}^{m\times n}$, and $\mathbf{b} \in \mathbb{R}^m$ are such that the matrix equation $A\mathbf{x} = \mathbf{b}$ has at exactly one solution.

(a) What can you say about the columns of the matrix A? Explain. (10 points)

(b) What is rank(A)? Prove that your answer is correct. (10 points)

3. Consider the matrix

$$A = \begin{bmatrix} -1 & -1 & 3 & 0 \\ 1 & -1 & 0 & 1 \\ 2 & 0 & -3 & -7 \end{bmatrix}$$

(a) Find a basis for nul(A). (8 points)

(b) Let H be the set of all vectors $\mathbf{b}$ such that the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution $\mathbf{x}$. Is H a subspace of $\mathbb{R}^3$? If so, find a basis for H. (12 points)

4. Suppose that $A \in \mathbb{M}^{m \times n}$ has entries a_{ij}, where $a_{ij} = ij$.

(a) Compute the column space of A.

(b) Find $\operatorname{rank}(A)$.

5.

(a) Write down the definition of *linear independence*. (This includes introducing the appropriate context, etc.) (5 points)

(b) Show that if $L : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation that is one-to-one, and $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a linearly independent set in $\mathbb{R}^n$, then the set $\{L(\mathbf{v}_1), \ldots, L(\mathbf{v}_p)\}$ is also linearly independent. (15 points)

6. Let $A \in \mathbb{M}^{n \times n}$. Assume that A^2 is equal to the zero matrix. Show that the matrix $I_n + A$ is invertible, and find its inverse. (20 points)