Math 16B, Section 1 **Spring 2006**
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REVIEW EXERCISES 3 – ANSWERS (USER BEWARE)

1. (a) $(0,0)$ is a saddle point, $\left(\frac{1}{12}, \frac{1}{6}\right)$ is a relative minimum.

   (b) $\left(\frac{1}{4}, \frac{3}{4}\right)$ is a relative minimum.

2. $a = 1,\ b = -\frac{1}{6}$.

3. $x = y = 18\frac{2}{3}$ inches, $z = 9\frac{1}{3}$ inches.

4. 6 units of labor, 2 units of capital.

5. (a) $\frac{1}{12}$ (b) $\frac{1}{2}$ (c) $2\ln 2 - \frac{3}{4}$ (d) $\frac{1}{4}$

6. (a) General solution: $y = 400,000 + Ce^{.065t}$
   Solution satisfying $y(0) = 300,000$: $y = 400,000 - 100,000e^{.065t}$

   (b) General solution: $y = e \pm \frac{1}{\sqrt{e^t + C}}$, and the constant function $y = e$
   Solution satisfying $y(0) = e - 1$: $y = e - e^{-t/2}$
   Solution satisfying $y(0) = e$: $y = e$

   (c) General solution: $y = \frac{1}{C - \sin^2 t}$, and the constant function $y = 0$
   Solution satisfying $y\left(\frac{\pi}{2}\right) = 1$: $y = \frac{1}{2 - \sin^2 t}$
   Solution satisfying $y\left(\frac{\pi}{2}\right) = 0$: $y = 0$

7. (a) $P' = .07P + 7000$

   (b) $P(t) = -100,000 + Ce^{.07t}$

   (c) $P(t) = -100,000 + 120,000e^{.07t}$

   (d) $P(18) = \$323,051$

8. (a) $p_2(x) = 10 + \frac{1}{20}(x - 100) - \frac{1}{8000}(x - 100)^2$

   (b) $p_2(104) = 10.198$; it differs from $\sqrt{104}$ by at most .0004.

9. (a) $p_3(x) = x - \frac{x^2}{2} + \frac{x^3}{3}$

   (b) $p_3(.1) = .09533 \approx \ln(1.1)$

   (c) The error in the approximation in (b) does not exceed .000025

10. (a) $p_3(x) = \frac{8}{3} + \frac{5}{2}(x - 1) + (x - 1)^2 + \frac{1}{6}(x - 1)^3$

    (b) $p_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

(c) In part (b), $p_3(x)$ is the given polynomial $f(x)$. In part (a), $p_3(x)$ is also $f(x)$, but written in terms of powers of $x-1$ instead of in terms of powers of x. In general, the nth Taylor polynomial at $x=a$ of a polynomial of degree n or less is just the given polynomial written in terms of powers of $x-a$.

11. (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n$ (b) 16

12. (a) $\dfrac{10}{11}$

(b) $p_3(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3$

(c) $p_3(1.1) = .909$; $\left|\dfrac{1}{1.1} - p_3(1.1)\right| \leq .0001$

(d) $\dfrac{10}{11} = \dfrac{1}{1.1}$. The Taylor polynomial $p_3(x)$ evaluated at $x = 1.1$ is thus an estimate of the number $.909090\ldots$, which turns out to be just a truncation of the infinite decimal.

13. (a) The probabilities are given in the following table.

2	3	4	5	6	7	8	9	10	11	12
1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

(b) $Pr(X < 7) = \dfrac{5}{12}$, $Pr(6 \leq x \leq 8) = \dfrac{4}{9}$, $Pr(x \text{ is odd}) = \dfrac{1}{2}$

(c) $\dfrac{1}{3}$ dollar

14. $E(X) = \dfrac{3}{5}$, $\mathrm{Var}(X) = \dfrac{12}{175}$

15. (a) $p_n = \dfrac{2}{5}\left(\dfrac{3}{5}\right)^n$ (b) $E(X) = \dfrac{2}{5}\displaystyle\sum_{n=1}^{\infty} n\left(\frac{3}{5}\right)^n$

(c) $\dfrac{x}{(1-x)^2} = \displaystyle\sum_{n=1}^{\infty} nx^n$, $-1 < x < 1$; $E(X) = \dfrac{3}{2}$