Math 16B, Section 1 **Spring 2006**
Sarason

REVIEW EXERCISES 2

1. A surveyor measures the angle of elevation of the top of a building as seen from a spot on the ground 100 feet from the building and finds it to be 1.25 radians. Estimate the height of the building.

2. In the Angst Amusement Park, the Ferris wheel has a radius of 20 feet and makes one complete revolution every half minute. What is the greatest vertical speed, in miles per hour, experienced by a passenger?

3. Dweeb Lake on the planet Nerdopia has a straight coastline. A boat on the lake travels parallel to the coast, at a distance of 1 nerdometer from the shore, at a constant speed of s nerdometers per nerdochrone. An observer on the shore measures the angle $\theta(t)$ between the shoreline and the line of sight from the observer to the ship. The measurements show that, at a certain time t_0, $\theta(t_0) = \pi/4$ radians and $\theta'(t_0) = -15$ radians per nerdochrone. What is the value of s?

4. Evaluate the integrals.

   (a) $\displaystyle\int_0^3 x^2 e^{-x^3}\,dx$ (b) $\displaystyle\int_0^3 x^3 e^{-x^2}\,dx$
   (c) $\displaystyle\int_1^e \frac{(\ln x)^2}{x^2}\,dx$ (d) $\displaystyle\int_0^\pi e^x \sin x\,dx$
   (e) $\displaystyle\int_0^\infty e^{-\sqrt{x}}\,dx$

5. Perform the integration $\displaystyle\int x\sqrt{x-1}\,dx$ both by substitution and by integration by parts. Reconcile the two answers.

6. For each part, find the general solution of the differential equation.

   (a) $(1+t^2)y' + ty = 0$ (b) $y' = -3y^2\sqrt{t}$
   (c) $y' = y \sin t \cos t$

7. (Midterm, F05) (a) Find the general solution of the differential equation $y' = t^2(y-1)^2$.
   (b) Find the solution satisfying the initial condition $y(0) = 0$.
   (c) Find the solution satisfying the initial condition $y(0) = 1$.

8. (a) Find the general solution of the differential equation $y' = -3y^2\sqrt{t}$.
   (b) Find the solution satisfying the initial condition $y(1) = 1$.
   (c) Find the solution satisfying the initial condition $y(1) = 0$.

9. (a) Find the most general function whose derivative equals its square.
   (b) Find the most general function that equals the square of its derivative.

10. (Final, F05) The Dinky University Molecular Biology Scholarship Fund starts the year with assets of $1,000,000, invested so as to earn interest at the rate of 10% per year, compounded continuously. The fund receives donations at the rate of $200,000 per year. Assume that donations are received continuously at the preceding rate, and that expenditures (grants plus expenses) occur continuously at the rate of A dollars per year.

    (a) Set up a differential equation satisfied by the DUMB Fund's assets $P(t)$ at time t (measured in years).
    (b) Find the general solution of the differential equation.
    (c) Find the solution satisfying the initial condition $P(0) = 1,000,000$.
    (d) What value must A have in order for the DUMB Fund's assets to be $1,000,000 at the end of the year.

11. Chili Conqueso buys a car for $40,000 with a 10% down payment and a loan for the remainder at 4.5% annual interest, compounded continuously. He will repay the loan with payments of $750 per month.

    (a) Assuming Chili's payments are made continuously, set up a differential equation satisfied by the unpaid amount $P(t)$ of the loan at time t, where t is measured in years, with $t = 0$ corresponding to the inception of the loan.
    (b) Find the general solution of the differential equation.
    (c) Find the function $P(t)$ for the case of Chili's loan.
    (d) Determine how long it will take Chili to repay the loan.

12. (Final, F04) In the kingdom of Xerotopia on the planet Nullphoria, the birth rate is 13 births per thousand inhabitants per year and the death rate is 14 deaths per thousand inhabitants per year. Suppose the kingdom gains inhabitants through immigration at the constant rate of A inhabitants per year.

    (a) Assuming as an approximation that births, deaths and immigration occur continuously, set up a differential equation satisfied by the population $P(t)$ of Xerotopia at time t.
    (b) Find the general solution of the differential equation.
    (c) Suppose the population of Xerotopia is now 40,000,000. How large must A be to guarantee that the future population will not fall below that number?