ANSWERS TO REVIEW EXERCISES 1 (USER BEWARE)

1. (a) $\frac{\partial f}{\partial x} = 3x(x^2 + y^2)^{1/2}$, $\frac{\partial f}{\partial y} = 3y(x^2 + y^2)^{1/2}$

    $\frac{\partial^2 f}{\partial x^2} = \frac{6x^2 + 3y^2}{(x^2+y^2)^{1/2}}$, $\frac{\partial^2 f}{\partial y^2} = \frac{3x^2 + 6y^2}{(x^2+y^2)^{1/2}}$, $\frac{\partial^2 f}{\partial x \partial y} = \frac{3xy}{(x^2+y^2)^{1/2}}$

    (b) $\frac{\partial f}{\partial x} = 2x\, e^{(x^2+y^2-z^2)}$, $\frac{\partial f}{\partial y} = 2y\, e^{(x^2+y^2-z^2)}$, $\frac{\partial f}{\partial z} = -2z\, e^{(x^2+y^2-z^2)}$

    $\frac{\partial^2 f}{\partial x^2} = (2 + 4x^2)e^{(x^2+y^2-z^2)}$, $\frac{\partial^2 f}{\partial y^2} = (2 + 4y^2)e^{(x^2+y^2-z^2)}$, $\frac{\partial^2 f}{\partial z^2} = (-2 + 4z^2)e^{(x^2+y^2-z^2)}$

    (c) $\frac{\partial f}{\partial x} = \frac{2x}{x^2+y^2}$, $\frac{\partial f}{\partial y} = \frac{2y}{x^2+y^2}$

    $\frac{\partial^2 f}{\partial x^2} = \frac{2y^2 - 2x^2}{(x^2+y^2)^2}$, $\frac{\partial^2 f}{\partial y^2} = \frac{2x^2 - 2y^2}{(x^2+y^2)^2}$, $\frac{\partial^2 f}{\partial x \partial y} = \frac{-4xy}{(x^2+y^2)^2}$.

2. For $x = 4$, $y = 2$:
    Marginal productivity of labor $= 12/5$
    Marginal productivity of capital $= 1/5$

3. (a) $(0,0)$, saddle point; $(-5, -5)$, relative minimum

    (b) $(-2, 1)$, relative minimum; $(-2, -1)$, relative maximum

    (c) $(0, 0)$, saddle point; $\left(\frac{1}{2}, \frac{1}{2}\right)$, relative minimum.

4. $a = 3$, $b = -1$.

5. Maximum $= 16\sqrt{2}$, attained at $(\sqrt{2}, \sqrt{2})$
    Minimum $= -16\sqrt{2}$, attained at $(-\sqrt{2}, -\sqrt{2})$

6. $x = 12$ inches, $y = 12$ inches, $z = 36$ inches

7. $40(15)^{1/4}$ units of labor, $40(15)^{-3/4}$ units of capital

$$\frac{\text{Labor costs}}{\text{Capital costs}} = 3.$$

8. (a) $I = -\frac{1}{3}$ (b) $I = -\frac{4}{9}$ (c) $I = \frac{2}{3}$