Math 16B, Section 1 **Spring 2006**
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REVIEW EXERCISES 1

1. In each part, find all first and second partial derivatives of the given function.
(a) $f(x, y) = (x^2 + y^2)^{3/2}$ (b) $f(x, y) = e^{(x^2+y^2-z^2)}$
(c) $f(x, y) = \ln(x^2 + y^2)$

2. (Midterm, F05) In the manufacture of Cuddly–Wuddly dolls, the number of dolls produced with the utilization of x units of labor and y units of capital is given by the production function $f(x, y) = \sqrt{6x^2 + y^2}$. Determine the marginal productivities of labor and capital when $x = 4$ and $y = 2$.

3. In each part, determine the critical points of the given function $f(x, y)$, and classify the critical points, to the extent possible, by means of the second-derivative test.
(a) $f(x, y) = x^3 + y^3 - 15xy$ (b) $f(x, y) = x^4 + y^3 + 32x - 3y$
(c) (Midterm, F05) $f(x, y) = 16x^5 + 5y^2 - 10xy$

4. For which values of a and b does the straight line $ax + b$ give the best least-squares approximation to the curve $y = x^2$ at the four points $(0, 0), (1, 1), (2, 4), (3, 9)$?

5. (Midterm, F04) Find the maximum and minimum values of the function $f(x, y) = 7x + 9y$ on the curve $7x^4 + 9y^4 = 64$.

6. According to U.S. postal regulations, a rectangular package whose three side lengths measure x inches, y inches, z inches, must satisfy $2x + 2y + z \leq 84$. What values of x, y, z give the dimensions of a package whose surface area, $2xy + 2xz + 2yz$, is maximum?

7. (Final, F05) The number of pairs of shoes the Phlim Zee Shoe Company can manufacture per week with the utilization of x units of labor and y units of capital is given by the production function $f(x, y) = 40x^{3/4}y^{1/4}$. Each unit of labor costs the company \$200 and each unit of capital costs it \$1,000. To manufacture 1,600 pairs of shoes per week at minimum cost, how many units of labor and how many units of capital should the company utilize? What is the corresponding ratio of labor costs to capital costs.

8. In each part, evaluate the integral I.

   (a) $I = \iint_R (x^4 - y^3)dxdy$, where R is the triangle with vertices $(0, 0), (-1, 1), (1, 1)$.

   (b) $I = \iint_R (y - x^2)dxdy$, where R is the region defined by the inequalities $0 \leq x \leq 1$, $y^2 \leq x^3$.

   (c) $I = \iint_R y\, dxdy$, where R is the region defined by the inequalities $x^2 + y^2 \leq 1$, $y \geq 0$.