REVIEW EXERCISES 2

1. Perform the differentiations. (a) $\dfrac{d}{dt}(\sin(\ln t))$ (b) $\dfrac{d}{dt}(\ln(\sin t))$ (c) $\dfrac{d}{dt}(\sec(e^t))$

   (d) $\dfrac{d}{dt}(\tan(\sec t))$

2. Evaluate the integrals. (a) $\displaystyle\int_1^e \frac{\ln x}{\sqrt{x}}\,dx$ (b) $\displaystyle\int_0^{\pi/2} x^2\cos x\,dx$ (c) $\displaystyle\int_0^{\pi/2} \cos^3 x\sin x\,dx$

   (d) $\displaystyle\int_0^{\pi/2} \cos^3 x\,dx$

3. Perform the integration $\displaystyle\int x\sqrt{x-1}\,dx$ both by substitution and by integration by parts. Reconcile the two answers.

4. The hypotenuse of a right triangle has length 13 meters and one side has length 5 meters. Find the tangent of the angle adjacent to that side and the tangent of the angle opposite that side.

5. A surveyor measures the angle of elevation of the top of a building as seen from a spot on the ground at a distance of 100 feet from the building and finds it to be 1.25 radians. Given that $\sin 1.25 \approx .95$ and $\cos 1.25 \approx .39$, estimate the height of the building.

6. The minute hand on the clock of the Vergessen Memorial Tower is 7 feet long. What is the vertical speed in feet per minute of the tip of the minute hand at 12:15p.m.?

7. In the Angst Amusement Park, the Ferris wheel has a radius of 20 feet and makes one complete revolution every half minute. What is the greatest vertical speed, in miles per hour, experienced by a passenger?

8. Angela Prissy drives a station wagon at a constant speed along a straight highway in the Utah salt flats. Officer Lester Luster mans radar observation post B located one mile from the highway. Let A be the closest point to B on the highway, $C = C(t)$ the location of the Prissy wagon at time t (measured in minutes), and $\theta = \theta(t)$ the angle at the vertex B in the right triangle ABC. When $\theta = \pi/3$ radians, Officer Luster measures θ' to be $3/8$ radians per minute. How fast is Angela driving?

9. Find the general solutions of the differential equations. (a) $y' = y\sin t\cos t$ (b) $y' = -y^3 t^3$
   (c) $y' - y\cos t = \cos t$

10. Solve the initial-value problems

    (a) $y' = y^2\sin t$, $y(0) = 1$

    (b) $y' = y^2\sin t$, $y(0) = 0$

    (c) $y' = .075y - 7,500$, $y(0) = 50,000$

11. (a) Find the general solution of the differential equation $y' = -3y^2\sqrt{t}$.

(b) Find the solution satisfying the initial condition $y(1) = 1$.

(c) Find the solution satisfying the initial condition $y(1) = 0$.

12. (a) Find the most general function whose derivative equals its square.

(b) Find the most general function that equals the square of its derivative.

13. Felicia La Groucha takes out a bank loan for $50,000 at an interest rate of 5% per year, compounded continuously. She will repay the loan with yearly payments of $12,000, applied continuously.

(a) Set up a differential equation satisfied by the unpaid amount $P(t)$ of the loan at time t.

(b) Solve the equation subject to the initial condition $P(0) = 50,000$.

14. Chili Conqueso buys a car for $40,000 with a 10% down payment and a loan for the remainder at 4.5% annual interest, compounded continuously. He will repay the loan with payments of $750 per month.

(a) Assuming Chili's payments are made continuously, set up a differential equation satisfied by the unpaid amount $P(t)$ of the loan at time t, where t is measured in years, with $t = 0$ corresponding to the inception of the loan.

(b) Find the general solution of the differential equation.

(c) Find the function $P(t)$ for the case of Chili's loan.

(d) Determine how long it will take Chili to repay the loan.