Math 16B, Section 1 **Fall 2005**
Sarason

REVIEW EXERCISES 1

1. In each part, find the first partial derivatives of the given function $f(x,y)$
   (a) $f(x,y) = x^5 - 10x^3y^2 + 5xy^4$ (b) $f(x,y) = 20x^{3/5}y^{2/5}$ (c) $f(x,y) = e^{-x^2/y}$

2. In each part, evaluate the integral $I = \iint_R f(x,y)dydx$ of the given function f over the given region R.

   (a) $f(x,y) = x^2 + y^4$; R is the rectangle given by $-1 \le x \le 1$, $0 \le y \le 2$.
   (b) $f(x,y) = x^2 - y^2$; R is the triangle with vertices $(0,0), (1,0), (1,2)$.
   (c) $f(x,y) = x^2y^3$; R is defined by the inequalities $x^2 + y^2 \le 1$, $y \ge 0$.

3. For each part, determine the critical points of the function $f(x,y)$, and determine the nature of each critical point, to the extent possible, by means of the second derivative test.

   (a) $f(x,y) = x^3 - y^2 - 3x + 4y$ (b) $f(x,y) = x^3 + y^3 - 9xy$
   (c) $f(x,y) = 4x^4 - 2y^2 - xy$ (d) $f(x,y) = 4x^8 - 8x^4 - xy$

4. (a) Find the maximum and minimum values of the function $f(x,y) = 7x + 9y$ on the curve $7x^4 + 9y^4 = 64$.

   (b) Where does the function $f(x,y) = x^2 + 3y^2 + 10$ attain its minimum value under the constraint $x + y = 8$?

   (c) Maximize the function $f(x,y,z) = 3x^{1/3}y^{1/3}z^{1/3}$ in the region $x > 0$, $y > 0$, $z > 0$ under the constraint $3x + 6y + 2z = 12$.

5. Let $(a_1,b_1), (a_2,b_2), \ldots, (a_N,b_N)$ be N distinct points in the (x,y)-plane. Find the point (x,y) in the plane that minimizes the sum of the squares of the distances from the N points.

6. Find the least-squares linear approximation to the function x^2 on the interval $[0,1]$, in other words, find the values of the constants a and b that minimize the integral

$$I(a,b) = \int_0^1 (x^2 - ax - b)^2 dx.$$

7. The Connecticut House of Elegant Apparel (CHEAP) wins a contract for its Central American subsidiary to provide 20,000 tee shirts for the Florida Anti-Intellectual League (FAIL). The number of shirts that can be produced with the utilization of x units of labor and y units of capital is given by the production function $f(x,y) = 500\ x^{4/5}y^{1/5}$. Each unit of labor costs 1000 pesos and each unit of capital costs 8000 pesos. How much labor and capital should CHEAP utilize so as to minimize the cost of production?

8. The Mexican firm Novedades Terminador SSA manufactures Arnold masks. The number of masks it can produce in a week with the utilization of x units of labor and y units of capital is given by the production function $f(x,y) = 500\ x^{3/5}y^{2/5}$. Each unit of labor costs 100 pesos per week and each unit of capital costs 200 pesos per week. Total expenses are limited to 1000 pesos per week. How many units of labor and capital should the firm utilize per week so as to maximize production?