MATH 113 HW 4: Subgroups, cyclic groups

Due in class on Thursday, September 28.

1. For each of the following statements, provide either a proof or a counterexample.

   (a) Let G be a group, and $a, b \in G$. If $(ab)^n = e$, then $(ba)^n = e$.

   (b) An element a of a group G has order n if and only if $a^n = e$ in G.

   (c) Every abelian group is cyclic.

   (d) Every cyclic group of order > 2 has at least two distinct generators.

2. Prove the following statements about subgroups.

   (a) If H is a non-empty finite subset of a group G and H is closed under the operation in G, then H is a subgroup of G.

   (b) If H and K are subgroups of a group G, then $H \cap K$ is a subgroup of G.

3. Prove that if $n \in \mathbb{Z}^+$, then $n^3 - n$ is divisible by 3.

4. Find all subgroups of $\mathbb{Z}_{60}$ and draw the subgroup diagram for $\mathbb{Z}_{60}$. (Note: It is okay for the lines to cross, as long as it is clear what the lines are connecting.)

5. Let $p, q \in \mathbb{Z}^+$ be two different prime numbers.

   (a) How many generators are there for the group $\mathbb{Z}_{pq}$? (Explain your answer.)

   (b) How many generators are there for the group $\mathbb{Z}_{p^2}$? (Explain your answer.)

6. Use the Euclidean Algorithm to find the greatest common divisor of the following pairs of integers.

   (a) 24 and 9. (b) 11391 and 5673. (c) 116 and -84.

No group problem this week. Study for the exam!