Worksheet PDP, Spring 2005

week 4

Arclength

1. Give a formula for the length of the curve given by $y = f(x)$, between $x = a$ and $x = b$.

2.

(a) Given the line segment connecting $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, what is the formula for computing the arc length of this curve?

(b) Use the arc length formula to compute the length of the curve $y = mx + b$ between $x = x_1$ and $x = x_2$. Does this correspond with your answer to (a)?

3. True or False? If $f(x) \geq g(x)$ for all x in $[a, b]$, then the length of the graph of f between a and b is greater than or equal to the length of the graph of g between a and b.

What if we replaced the assumption by $f'(x) \geq g'(x)$?

4. Give a formula for the length of the curve given by $x = g(y)$, between $y = a$ and $y = b$.

5. Find the following arc lengths:

(a) $f(x) = 2\cosh(x) = e^x + e^{-x}$, with $-3 \leq x \leq 3$ (this is a catenary, the shape of a wire hanging from two points).

(b) $f(t) = t^3 + \frac{1}{12t}$, with $0 \leq t \leq x$.

(c) $f(x) = \ln(\cos(x))$, with $0 \leq x \leq \frac{1}{2}\pi$.

(d) $y^2 = 4x$, with $0 \leq y \leq 2$.

(e) $y = e^x$, with $0 \leq x \leq 1$.

(f) $f(t) = t^2$, with $0 \leq t \leq x$.

(g) $h(x) = \sqrt{x}(1 - x/3)$, with $1 \leq x \leq 9$.

6. True or False: The arc length of $y = cf(x)$ between $x = a$ and $x = b$ is c times the arc length of $y = f(x)$ between $x = a$ and $x = b$.

7. Suppose $h_1(t)$ and $h_2(t)$ are two functions with continuous derivatives on $[a, b]$, satisfying $h_1'(t) \cdot h_2'(t) = \frac{1}{4}$. Then what is the length of the curve given by

$f(t) = h_1(t) - h_2(t)$ between $x = a$ and $x = b$?

Area of a surface of revolution

8. What is the surface area of a cilinder of height l and radius r? What is the surface area of a cone with slant height l and base radius r?

9. What is the formula for the area of the surface obtained by revolving the curve $y = f(x)$ from $x = a$ to $x = b$ around the x-axis? What if the curve was given by $x = g(y)$?

10. What is the formula for the area of the surface obtained by revolving the curve $y = f(x)$ from $y = c$ to $y = d$ around the y-axis? What if the curve was given by $x = g(y)$?

Write your answers to the last two problems in the following table.

given formula	$y = f(x)$	$x = g(y)$	either way
given interval	$a \le x \le b$	$c \le y \le d$	either
around x-axis	$\int_a^b \qquad dx$	$\int_c^d \qquad dy$	$\int \qquad ds$
around y-axis	$\int_a^b \qquad dx$	$\int_c^d \qquad dy$	$\int \qquad ds$

11. Set up two integrals to find the area of the surface obtained by revolving the curve around the right axis. Evaluate at least one.

1. $y = \sqrt{x}$, x-axis from $x = 2$ to $x = 4$.
2. $y = x^3$, x-axis from $x = 1$ to $x = 7$.
3. $y = \cos(x)$, x-axis from $x = 0$ to $x = \pi/4$.
4. $x^3/6 + 1/(2x)$, x-axis from $x = 1/2$ to $x = 1$.
5. $x = 1 + y^2$, x-axis from $y = 1$ to $y = 2$.
6. $y = 1 - x^2$, y-axis from $x = -1$ to $x = 0$.
7. $2x = e^y + e^{-y}$, y-axis from $y = -1$ to $y = 1$.
8. $x^2 + y^2 = 4$, y-axis from $x = -2$ to $x = 2$ (careful! make a picture)

12. If the curve $y = f(x)$ between $x = a$ and $x = b$ is rotated around the vertical line $x = c$, with $c < a$, find a formula for the area of the resulting surface.

13. Sketch the graph of $y = 1/x$ for $x \ge 1$.

1. Show that the region under the graph has infinite area.
2. Show that if the graph is rotated around the x-axis, the area of the resulting surface is infintite.
3. Is the volume inside the surface infinite or finite?