Worksheet 2, Spring 2005

January 21

1.

1. Can you write down a function whose derivative is $f(x)g'(x) + f'(x)g(x)$?
2. Can you write down all?
3. Using the above, rewrite
$$\int (f(x)g'(x) + f'(x)g(x))\ dx$$
4. Give a formula for $\int f'(x)g(x)\,dx$.
5. If you set $u = g(x)$ and $v = f(x)$, then $dv = f'(x)\,dx$. Now rewrite the formula you just found in terms of u and v, i.e., $\int u\,dv = \ldots$

2. Find the following integrals.

1. $\int x\cos(x)\,dx$
2. $\int \ln x\,dx$
3. $\int \arctan(x)\,dx$
4. $\int x^2 e^{3x}\,dx$
5. $\int \frac{\ln x}{x}\,dx$
6. $\int e^x \sin(2x)\,dx$
7. $\int \frac{\ln x}{x^2} dx$
8. $\int \cos(\ln x) dx$
9. $\int_0^1 x5^x dx$

3. Using integration by parts in the following integrals, what would you choose for $f'(x)$ and what for $g(x)$? (You don't have to do the integral)

1. $\int (x^3 + 2x - 3)\sin(x)\,dx$
2. $\int (\ln x)^4\,dx$
3. $\int x^5 e^{7x}\,dx$
4. $\int \cos(x)\sin(x)\,dx$

4. Find the volume generated by rotating the area enclosed by $f(x) = xe^x$, $y = 0$ and $x = 1$ around the x-axis.

5. Find 3 different ways to integrate $\int \sin x \cos x dx$.

6. **7.** Do the following integrals

1. $\int \sin^3 x \cos^2 x dx$
2. $\int \tan^4 x \sec^4 x dx$
3. $\int \tan^3(x)\cos^5(x)\,dx$
4. $\int \sin^3(x)\sec^3(x)\,dx$
5. $\int \tan^3(x)\sec^4(x)\,dx$ (twice)

8. Do the integral $\int \sec x\,dx$.

9. a. Prove the reduction formula for $n \geq 2$

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x dx$$

b. Evaluate $\int \cos^2 x dx$ and $\int \cos^4 x dx$.

10.

1. Show that for any (differentiable) function f we have

$$\int (f(x)e^x)\,dx = f(x)e^x - \int (f'(x)e^x)\,dx.$$

2. Repeat the above to show that

$$\int (f(x)e^x)\,dx = (f(x) - f'(x))e^x + \int (f''(x)e^x)\,dx.$$

3. If f is a polynomial of degree n what is $f^{(n+1)}(x)$?
4. Repeat the above to show that for a polynomial of degree n we have

$$\int (f(x)e^x)\,dx = \left(f(x) - f'(x) + f''(x) - \cdots + (-1)^{?} f^{(n)}(x)\right)e^x.$$

Should "?" be n or $n+1$?