

Worksheet 1, Spring 2005

January 19

1. Give an equation for the line tangent to the graph of f at $(a, f(a))$.
2. What does an antiderivative have to do with the area under the graph of a function? Use equations to make your answer precise and explain what these equations say.
3. Give a table of the derivative and antiderivative of the following functions.
 1. $\cos x$
 2. $\cos(3x + 1)$
 3. e^{4x}
 4. x^n
 5. x^{-n}
 6. \sqrt{x}
 7. $x^2\sqrt{x}$
 8. $1/x$
 9. $1/(x^2\sqrt{x})$
4. What form do you have to recognize in an integral to be able to use substitution?
5. Find
 1. $\int x \cos(x^2) dx$
 2. $\int \cos(x) \sin(x) dx$
 3. $\int \frac{\ln x}{x} dx$
 4. $\int x^2 e^{x^3} dx$
 5. $\int \cos^2(x) \sin(x) dx$
 6. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

6.

1. Can you write down a function whose derivative is $f(x)g'(x) + f'(x)g(x)$?
2. Can you write down all?
3. Using the above, rewrite

$$\int (f(x)g'(x) + f'(x)g(x)) dx$$

4. Give a formula for $\int f'(x)g(x) dx$.
 5. If you set $u = g(x)$ and $v = f(x)$, then $dv = f'(x) dx$. Now rewrite the formula you just found in terms of u and v , i.e., $\int u dv = \dots$
7. Find the following integrals.

1. $\int x \cos(x) dx$
2. $\int \ln x dx$
3. $\int \arctan(x) dx$
4. $\int x^2 e^{3x} dx$
5. $\int \frac{\ln x}{x} dx$
6. $\int e^x \sin(2x) dx$

8. Using integration by parts in the following integrals, what would you choose for $f'(x)$ and what for $g(x)$? (You don't have to do the integral)

1. $\int (x^3 + 2x - 3) \sin(x) dx$
2. $\int (\ln x)^4 dx$
3. $\int x^5 e^{7x} dx$
4. $\int \cos(x) \sin(x) dx$

9.

1. Show that for any (differentiable) function f we have

$$\int (f(x)e^x) dx = f(x)e^x - \int (f'(x)e^x) dx.$$

2. Repeat the above to show that

$$\int (f(x)e^x) dx = (f(x) - f'(x))e^x + \int (f''(x)e^x) dx.$$

3. If f is a polynomial of degree n what is $f^{(n+1)}(x)$?
4. Repeat the above to show that for a polynomial of degree n we have

$$\int (f(x)e^x) dx = \left(f(x) - f'(x) + f''(x) - \dots + (-1)^n f^{(n)}(x) \right) e^x.$$

Should "n" be n or $n + 1$?