Math 115 Professor Kenneth A. Ribet

Homework assignment #13
Due December 1, 2006

I understand now that the good way to do problem #5 is to reduce the equation $a^4 + b^4 = 2c^2$ to another problem of the type that we have studied already. The aim is to prove that $a^4 = b^4$; this leads one to consider the difference $a^4 - b^4$. Since $a^4 + b^4$ is even, $a^4 - b^4$ is even as well; thus we can let x be defined by $2x = a^4 - b^4$. We'd like to prove that x is 0; if we want to argue by contradiction, we can suppose that x is non-zero. After re-ordering a and b if necessary, we can suppose that x is positive.

Show that $c^4 - x^2$ is a non-zero perfect fourth power, say d^4. Then we have $c^4 - d^4 = x^2$, with all of c, d and x non-zero. The equation $c^4 = d^4 + x^2$ is similar to one that we studied in class. Now "all" we have to do is to solve this equation as we solved the equation in class or perhaps reduce this equation to the one we had in class.