SOLUTIONS TO HOMEWORK #7, MATH 54 SECTION 001, SPRING 2012

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Beware of typos. These may not be the only ways to solve these problems. In fact, these may not even be the best ways to solve these problems. Congratulations to you if you find better solutions!

1. Ex. 5.2.6: *Find the characteristic polynomial and the eigenvalues of the matrices in Exercises 1-8.* $\begin{bmatrix} 3 & -4 \\ 4 & 8 \end{bmatrix}$

Solution. Let A be the matrix in the problem statement. Then the characteristic polynomial of A is:

$$|A-\lambda I| = \begin{vmatrix} 3-\lambda & -4 \\ 4 & 8-\lambda \end{vmatrix} = \boxed{(3-\lambda)(8-\lambda)-(-4)4} = \boxed{\lambda^2 - 11\lambda + 40}.$$

From the quadratic formula, the eigenvalues of A are:

$$\frac{11 \pm \sqrt{121-160}}{2} = \boxed{\frac{11 \pm \sqrt{39}i}{2}}.$$

"$\boxed{\text{There are no real eigenvalues}}$" is also an acceptable answer. □

2. Ex. 5.2.12: *Exercises 9-14 require techniques from Section 3.1. Find the characteristic polynomial of each matrix, using either a cofactor expansion or the special formula for 3×3 determinants described in Exercises 15-18 in Section 3.1. [*Note: *Finding the characteristic polynomial of a 3×3 matrix is not easy to do with just row operations, because the variable λ is involved.]* $\begin{bmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

Solution. Let A be the matrix in the problem statement. Then the characteristic polynomial of A is:

$$|A-\lambda I| = \begin{vmatrix} -1-\lambda & 0 & 1 \\ -3 & 4-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (4-\lambda)\begin{vmatrix} -1-\lambda & 1 \\ 0 & 2-\lambda \end{vmatrix} = \boxed{(4-\lambda)(-1-\lambda)(2-\lambda)}.$$ □

3. Ex. 5.2.17: *For the matrices in Exercises 15-17, list the eigenvalues, repeated according to their multiplicities.* $\begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ -5 & 1 & 0 & 0 & 0 \\ 3 & 8 & 0 & 0 & 0 \\ 0 & -7 & 2 & 1 & 0 \\ -4 & 1 & 9 & -2 & 3 \end{bmatrix}$

Solution. The eigenvalues of a triangular matrix with multiplicities are the entries along its main diagonal, with multiplicities. The answer is $\boxed{0, 1, 1, 3, 3}$. □

4. Ex. 5.2.18: *It can be shown that the algebraic multiplicity of an eigenvalue λ is always greater than or equal to the dimension of the eigenspace corresponding to λ. Find h in the matrix A below such that the eigenspace for $\lambda = 5$ is two-dimensional: $A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$*

Solution. Let B equal:

$$A - 5I = \begin{bmatrix} 0 & -2 & 6 & -1 \\ 0 & -2 & h & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & -4 \end{bmatrix},$$

and let $\mathbf{b}_1, \ldots, \mathbf{b}_4$ be the columns of B.

Then the eigenspace for 5 is $\operatorname{Nul} B$, so we want to find all h for which $\dim \operatorname{Nul} B = 2$. From the rank-nullity theorem, this is the same as needing $\dim \operatorname{Col} B = 4 - 2 = 2$.

Because $\mathbf{b}_2$ and $\mathbf{b}_4$ are nonzero and not constant multiples of each other, $\{\mathbf{b}_2, \mathbf{b}_4\}$ is linearly independent. Then $\dim \operatorname{Col} B$ will be 2 if and only if $\{\mathbf{b}_2, \mathbf{b}_4\}$ is a basis of $\operatorname{Col} B$, if and only if $\{\mathbf{b}_2, \mathbf{b}_4\}$ spans $\operatorname{Col} B$. Since $\mathbf{b}_1$ is already in $\operatorname{Span}\{\mathbf{b}_2, \mathbf{b}_4\}$, we only need to find those h for which $\mathbf{b}_3$ is in $\operatorname{Span} \operatorname{Col} B$, i.e. when there are constants x_1, x_2 for which:

$$x_1\mathbf{b}_2 + x_2\mathbf{b}_4 = \mathbf{b}_3.$$

Rewriting this in matrix form, we have:

$$\begin{bmatrix} -2 & -1 & 6 \\ -2 & 0 & h \\ 0 & 4 & 0 \\ 0 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} -2 & -1 & 6 \\ 0 & 1 & h-6 \\ 0 & 4 & 0 \\ 0 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} -2 & -1 & 6 \\ 0 & 4 & 0 \\ 0 & 1 & h-6 \\ 0 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} -2 & -1 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & h-6 \\ 0 & 0 & 0 \end{bmatrix}.$$

Therefore, there are constants x_1, x_2 for which $x_1\mathbf{b}_2 + x_2\mathbf{b}_4 = \mathbf{b}_3$ if and only if $\boxed{h = 6}$, and tracing back through gives that this is the final answer. □

5. Ex. 5.2.22: *In Exercises 21 and 22, A and B are $n \times n$ matrices. Mark each statement True or False. Justify each answer.*

a. *If A is 3×3, with columns $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, then $\det A$ equals the volume of the parallelepiped determined by $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$.*

b. $\det A^T = (-1)\det A$.

c. *The multiplicity of a root r of the characteristic equation of A is called the algebraic multiplicity of r as an eigenvalue of A.*

d. *A row replacement operation on A does not change the eigenvalues.*

Solution. a. $\boxed{\text{False}}$. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. Then $\det A = -1 < 0$, and so cannot be the volume of any shape in $\mathbb{R}^3$ at all. (The correct statement is that $|\det A|$ is the volume of the parallelepiped determined by $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$.)

b. $\boxed{\text{False}}$. Let $A = I$. Then $\det A^T = \det A = 1$, but $(-1)\det A = (-1)1 = -1$. (The correct rule is that $\det A^T = \det A$.)

c. $\boxed{\text{True}}$. This the definition of algebraic multiplicity as found on p.282, just before Example 4 of Section 5.2.

d. $\boxed{\text{False}}$. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, and $B = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Then since A is triangular, its eigenvalues are the entries on the main diagonal, i.e. 1 and 2. Likewise, 1 is the only eigenvalue of B. Therefore, A and B have different eigenvalues, even though B can be obtained from A by a single row replacement operation. □

6. Ex. 5.3.3: *In Exercises 3 and 4, use the factorization $A = PDP^{-1}$ to compute A^k, where k represents an arbitrary positive integer.*

$$\begin{bmatrix} a & 0 \\ 3(a-b) & b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}.$$

Solution.

$$A^k = PD^kP^{-1} = \boxed{\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} a^k & 0 \\ 0 & b^k \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}} = \begin{bmatrix} a^k & 0 \\ 3a^k & b^k \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} a^k & 0 \\ 3(a^k - b^k) & b^k \end{bmatrix}}.$$ □

7. Ex. 5.3.6: *In Exercises 5 and 6, the matrix A is factored in the form PDP^{-1}. Use the Diagonalization Theorem to find the eigenvalues of A and a basis for each eigenspace.*

$$\begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 4 \\ -1 & 0 & -2 \end{bmatrix}.$$

Solution. By the diagonalization theorem, the eigenvalues of A are $\boxed{5 \text{ and } 4}$. Bases for the eigenspaces of 5 and 4 are $\boxed{\left\{ \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}}$ and $\boxed{\left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \right\}}$, respectively. □

8. Ex. 5.3.18: *Diagonalize the matrices in Exercises 7-20, if possible... For Exercise 18, one eigenvalue is $\lambda = 5$ and one eigenvector is $(-2, 1, 2)$.* $\begin{bmatrix} -7 & -16 & 4 \\ 6 & 13 & -2 \\ 12 & 16 & 1 \end{bmatrix}$

Solution. Let A be the matrix in the problem statement. We are given that $(-2, 1, 2)$ is an eigenvector of A. We compute:

$$\begin{bmatrix} -7 & -16 & 4 \\ 6 & 13 & -2 \\ 12 & 16 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 14 - 16 + 8 \\ -12 + 13 - 4 \\ -24 + 16 + 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ -6 \end{bmatrix} = (-3) \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}.$$

Therefore, -3 is the eigenvalue of A corresponding to $(-2, 1, 2)$. We are also given that 5 is an eigenvalue of A.

Since we need to find the eigenspaces of 5 and -3 anyway, we will start by computing them:

$$\begin{bmatrix} A - 5I & \mathbf{0} \end{bmatrix} = \begin{bmatrix} -12 & -16 & 4 & 0 \\ 6 & 8 & -2 & 0 \\ 12 & 16 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & 4 & -1 & 0 \\ 6 & 8 & -2 & 0 \\ 12 & 16 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore, if $(A - 5I)\mathbf{x} = \mathbf{0}$, then x_2 and x_3 are free and $x_1 = -\frac{4}{3}x_2 + \frac{1}{3}x_3$. Thus:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -\frac{4}{3} \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} \frac{1}{3} \\ 0 \\ 1 \end{bmatrix}.$$

Therefore, $\lambda = 5$ has two associated linearly independent eigenvectors, for example $(4, -3, 0)$ and $(1, 0, 3)$. We also know that $\lambda = -3$ has $(-2, 1, 2)$ as an associated eigenvector. This gives three linearly independent eigenvectors, so $\boxed{A \text{ is diagonalizable}}$, and by the Diagonal Matrix theorem:

$$\boxed{\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -2 \\ -3 & 0 & 1 \\ 0 & 3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} -7 & -16 & 4 \\ 6 & 13 & -2 \\ 12 & 16 & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 \\ -3 & 0 & 1 \\ 0 & 3 & 2 \end{bmatrix}}.$$

□

9. Ex. 5.3.22: *In Exercises 21 and 22, A, B, P, and D are $n \times n$ matrices. Mark each statement True or False. Justify each answer. (Study Theorems 5 and 6 and the examples in this section carefully before you try these exercises.)*

a. *A is diagonalizable if A has n eigenvectors.*
b. *If A is diagonalizable, then A has n distinct eigenvalues.*
c. *If $AP = PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A.*
d. *If A is invertible, then A is diagonalizable.*

Solution. a. $\boxed{\text{False.}}$ As was shown in Example 4, $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ is not diagonalizable, but it does have $\lambda = 1$ as an eigenvalue, with associated eigenvector $(1, -1, 1)$. But then $(2, -2, 2)$ and $(-1, 1, -1)$ are also eigenvectors of A, so A has three different eigenvectors. (One possible corrected statement is "A is diagonalizable if A has n linearly independent eigenvectors.")

b. $\boxed{\text{False.}}$ Let $A = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Then because A is diagonal, A is diagonalizable (since $A = I^{-1}AI$). However, since A is triangular, its eigenvalues are the entries on the main diagonal, so A has only one eigenvalue. (One possible corrected statement is "If A has n distinct eigenvalues, then A is diagonalizable.")

c. $\boxed{\text{True.}}$ (Warning: The problem did not say that we could assume P is invertible! My solution will not assume it.) Let $\mathbf{p}_1, \dots, \mathbf{p}_n$ be the n columns of P, and let $d_1, \dots, d_n$ be the diagonal entries of D. Then the ith column of AP is $A\mathbf{p}_i$, while the ith column of PD is $d_i\mathbf{p}_i$. Since $AP = PD$, we have $A\mathbf{p}_i = d_i\mathbf{p}_i$. Therefore, if $\mathbf{p}_i \neq \mathbf{0}$, then $\mathbf{p}_i$ is an eigenvector of A with associated eigenvalue d_i.

d. $\boxed{\text{False.}}$ As was shown in Example 4, $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ is not diagonalizable. However, its determinant is:

$$2\begin{bmatrix} -6 & -3 \\ 3 & 1 \end{bmatrix} - 4\begin{bmatrix} -4 & -3 \\ 3 & 1 \end{bmatrix} + 3\begin{bmatrix} -4 & -6 \\ 3 & 3 \end{bmatrix} = 2 \cdot 3 - 4 \cdot 5 + 3 \cdot 6 = 4 \neq 0,$$

and so is invertible. (The matrix $\begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$ for any $a \neq 0$ is the simplest counterexample I could think of.) $\square$

10. Ex. 5.4.4: *Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be a basis for a vector space V and $T\colon V \to \mathbb{R}^2$ be a linear transformation with the property that*

$$T(x_1\mathbf{b}_1 + x_2\mathbf{b}_2 + x_3\mathbf{b}_3) = \begin{bmatrix} 2x_1 - 4x_2 + 5x_3 \\ -x_2 + 3x_5 \end{bmatrix}$$

Find the matrix for T relative to $\mathcal{B}$ and the standard basis for $\mathbb{R}^2$.

Solution. For any vector $\mathbf{v}$ in $\mathbb{R}^2$, the coordinate representation of $\mathbf{v}$ relative to the standard basis of $\mathbb{R}^2$ is $\mathbf{v}$ itself. Therefore, the formula for the matrix of T relative to $\mathcal{B}$ and the standard basis for $\mathbb{R}^2$ simplifies to:

$$\begin{bmatrix} T(\mathbf{b}_1) & T(\mathbf{b}_2) & T(\mathbf{b}_3) \end{bmatrix} = \boxed{\begin{bmatrix} 2 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}}. \qquad \square$$

11. Ex. 5.4.6: *Let $T\colon \mathbb{P}_2 \to \mathbb{P}_4$ be the transformation that maps a polynomial $\mathbf{p}(t)$ into the polynomial $\mathbf{p}(t) + t^2\mathbf{p}(t)$.*

a. *Find the image of $\mathbf{p}(t) = 2 - t + t^2$.*
b. *Show that T is a linear transformation.*
c. *Find the matrix for T relative to the bases $\{1, t, t^2\}$ and $\{1, t, t^2, t^3, t^4\}$.*

Solution. a. $T(2 - t + t^2) = (2 - t + t^2) + t^2(2 - t + t^2) = \boxed{2 - t + 3t^2 - t^3 + t^4}$.

b. Let $\mathbf{p}$ and $\mathbf{q}$ be any polynomials in $\mathbb{P}_2$, and let c be any real scalar. Then:

$$T(\mathbf{p} + \mathbf{q}) = (\mathbf{p} + \mathbf{q}) + t^2(\mathbf{p} + \mathbf{q}) = (\mathbf{p} + t^2\mathbf{p}) + (\mathbf{q} + t^2\mathbf{q}) = T(\mathbf{p}) + T(\mathbf{q})$$
$$T(c\mathbf{p}) = (c\mathbf{p}) + t^2(c\mathbf{p}) = c(\mathbf{p} + t^2\mathbf{p}) = cT(\mathbf{p}),$$

which is what it means for T to be linear.

c. Let $\mathcal{C}$ be the basis $\{1, t, t^2, t^3, t^4\}$ of $\mathbb{P}_4$. Then the matrix of T relative to $\{1, t, t^2\}$ and $\mathcal{C}$ is:

$$\begin{bmatrix} [T(1)]_{\mathcal{C}} & [T(t)]_{\mathcal{C}} & [T(t^2)]_{\mathcal{C}} \end{bmatrix} = \begin{bmatrix} [1 + t^2]_{\mathcal{C}} & [t + t^3]_{\mathcal{C}} & [t^2 + t^4]_{\mathcal{C}} \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}. \qquad \square$$

12. Ex. 5.4.21: *Verify the statements in Exercises 19-24. The matrices are square.*
If B is similar to A and C is similar to A, then B is similar to C.

Solution. Since B is similar to A, there is an invertible matrix P with $P^{-1}AP = B$. Multiplying on the left by P and on the right by P^{-1} gives

$$A = PBP^{-1}.$$

Since C is similar to A, there is an invertible matrix Q with $Q^{-1}AQ = C$, and plugging in $A = PBP^{-1}$ gives:

$$Q^{-1}PBP^{-1}Q = C.$$

Then $P^{-1}Q$ is invertible, and its inverse is $Q^{-1}(P^{-1})^{-1} = Q^{-1}P$. Therefore, $(P^{-1}Q)^{-1}B(P^{-1}Q) = C$, so B is similar to C. □

13. Ex. 5.4.22: *If A is diagonalizable and B is similar to A, then B is also diagonalizable.*

Solution. Since A is diagonalizable, there is a diagonal matrix D for which D is similar to A. Then by the previous exercise, D is similar to B, so B is diagonalizable. □

14. Ex. 5.5.4: *Let each matrix in Exercises 1-6 act on $\mathbb{C}^2$. Find the eigenvalues and a basis for each eigenspace in $\mathbb{C}^2$.* $\begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$

Solution. Let A be the matrix in the problem statement. Then the characteristic polynomial of A is:

$$\det A - \lambda I = \begin{vmatrix} 5-\lambda & -2 \\ 1 & 3-\lambda \end{vmatrix} = (5-\lambda)(3-\lambda) - 1(-2) = \lambda^2 - 8\lambda + 17.$$

Its roots, i.e. the eigenvalues of A are:

$$\frac{8 \pm \sqrt{64-68}}{2} = \boxed{4 \pm i}.$$

Because $4+i$ is an eigenvalue of A, $A-(4+i)I$ must have row rank 1, so:

$$A - (4+i)I = \begin{bmatrix} 1-i & -2 \\ 1 & -1-i \end{bmatrix} \sim \begin{bmatrix} 1-i & -2 \\ 0 & 0 \end{bmatrix}.$$

Therefore, $\boxed{\left\{\begin{bmatrix} 2 \\ 1-i \end{bmatrix}\right\}}$ is a (complex) basis of the eigenspace of A corresponding to $4+i$. Since A is real, $\boxed{\left\{\begin{bmatrix} 2 \\ 1+i \end{bmatrix}\right\}}$ is a (complex) basis of the eigenspace of A corresponding to $4-i$. □

15. Ex. 5.5.6: $\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}$

Solution. Let A be the matrix in the problem statement. Then the characteristic polynomial of A is:

$$\det A - \lambda I = \begin{vmatrix} 4-\lambda & 3 \\ -3 & 4-\lambda \end{vmatrix} = (4-\lambda)^2 - 3^2 = (\lambda-4)^2 - 3^2.$$

Its roots, i.e. the eigenvalues of A are therefore $\boxed{4 \pm 3i}$.

$$A - (4+3i)I = \begin{bmatrix} -3i & 3 \\ -3 & -3i \end{bmatrix} \sim \begin{bmatrix} i & -1 \\ -3 & -3i \end{bmatrix} \sim \begin{bmatrix} i & -1 \\ 0 & 0 \end{bmatrix}.$$

Therefore, $\boxed{\left\{\begin{bmatrix} 1 \\ i \end{bmatrix}\right\}}$ is a (complex) basis of the eigenspace of A corresponding to $4+3i$. Since A is real, $\boxed{\left\{\begin{bmatrix} 1 \\ -i \end{bmatrix}\right\}}$ is a (complex) basis of the eigenspace of A corresponding to $4-3i$. □

16. Ex. 5.5.14: *In Exercises 13-20, find an invertible matrix P and a matrix C of the form* $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ *such that the given matrix has the form $A = PCP^{-1}$. For Exercises 13-16, use information from Exercises 1-4.*
$\begin{bmatrix} 5 & -5 \\ 1 & 1 \end{bmatrix}$

Solution. Let A be the matrix in the problem statement. Then the characteristic polynomial of A is:

$$\det A - \lambda I = \begin{vmatrix} 5-\lambda & -5 \\ 1 & 1-\lambda \end{vmatrix} = (5-\lambda)(1-\lambda) - (-5)1 = \lambda^2 - 6\lambda + 10.$$

Its roots, i.e. the eigenvalues of A are:

$$\frac{6 \pm \sqrt{36-40}}{2} = 3 \pm i.$$

Because $3-i$ is an eigenvalue of A, $A-(3-i)I$ must have row rank 1, so:

$$A - (3-i)I = \begin{vmatrix} 2+i & -5 \\ 1 & -2+i \end{vmatrix} \sim \begin{bmatrix} 2+i & -5 \\ 0 & 0 \end{bmatrix}.$$

Therefore, $\mathbf{v} = \begin{bmatrix} 5 \\ 2+i \end{bmatrix}$ is a complex eigenvalue of A corresponding to $\lambda = 3-i$.

By Theorem 9, if:

$$P = \begin{bmatrix} \operatorname{Re}\mathbf{v} & \operatorname{Im}\mathbf{v} \end{bmatrix} = \boxed{\begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}}, \qquad C = \boxed{\begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}},$$

then P is invertible and $A = PCP^{-1}$, as needed. □

17. Ex. 5.5.16: $\begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$

Solution. By my solution to Ex. 5.5.4, $4-i$ is an eigenvalue of this matrix, with corresponding eigenvector $\begin{bmatrix} 2 \\ 1+i \end{bmatrix}$. Then by Theorem 9, if $\boxed{P = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}}$ and $\boxed{C = \begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix}}$ then P is invertible and $A = PCP^{-1}$. □