Solution to Homework 6

1. (4.5.6)
$$\left\{\begin{pmatrix} 6 \\ -2 \\ 5 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ 3 \\ 1 \end{pmatrix}\right\}$$
Dimension=2.

2. (4.5.8)
$$\left\{\begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}\right\}$$
Dimension=3.

3. (4.5.12) Dim=3, because the 1st, 2nd and 4th vector are linearly independent and the 3rd is a combination of the first 2.

4. (4.5.18)
$$\dim \operatorname{Nul} A = 1, \dim \operatorname{Col} A = 2.$$
A basis for Nul A is {(0,0,1)}. A basis for Col A is {(1,0,0),(4,7,0)}

5. (4.5.22) They span $\mathbb{P}_3$ because $1, t, t^2, t^3$ are all in their span. As they are linearly independent, they form a basis.

6. (4.5.27) Because for any n, $1, t, t^2, \ldots, t^n$ are linearly independent, so $\dim \mathbb{P} > n$ for any n. In other words, $\mathbb{P}$ is a space not spanned by a finite set.

7. (4.6.2) $\operatorname{Rank} A = 3, \dim \operatorname{Nul} A = 2$.

   Basis for $\operatorname{Col} A$:
$$\left\{\begin{pmatrix} 1 \\ -2 \\ -3 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ -6 \\ -6 \\ 4 \end{pmatrix}, \begin{pmatrix} 9 \\ -10 \\ -3 \\ 0 \end{pmatrix}\right\}$$
   Basis for $\operatorname{Row} A$:
$$\left\{\begin{pmatrix} 1 & -3 & 0 & 5 & -7 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 2 & -3 & 8 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 & 5 \end{pmatrix}\right\}$$
   Basis for $\operatorname{Nul} A$:
$$\left\{\begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -10 \\ 0 \\ 3 \\ 2 \\ 0 \end{pmatrix}\right\}$$

8. (4.6.6) $\operatorname{Rank} A^T = 3, \dim \operatorname{Nul} A = 0, \dim \operatorname{Row} A = 3$.

9. (4.6.16) 0.

10. (4.6.18)

    (a) F. Pivot columns of A form a basis of $\operatorname{Col} A$.

    (b) F. Among the columns of A.

    (c) T. $\dim \operatorname{Nul} A =$ number of free variables.

    (d) T.

    (e) T.

11. (4.6.33) In this case any column is a multiple of $\mathbf{u}$. Form $\mathbf{v}$ by writing down the multiplicities. If the first column is $\mathbf{0}$ just let $\mathbf{u}$ to be the first non-zero column.

12. (4.7.4) (i). Think about the case where $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ is just the standard basis.

13. (4.7.10) The one from $\mathcal{B}$ to C is $\begin{pmatrix} 8 & 3 \\ -5 & -2 \end{pmatrix}$ and the one from C to $\mathcal{B}$ is $\begin{pmatrix} 2 & 3 \\ -5 & -8 \end{pmatrix}$

14. (4.7.14) The change-of-coordinates matrix is $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ -3 & -5 & 0 \end{pmatrix}$ and

$$t^2 = 3(1 - 3t^2) - 2(2 + t - 5t^2) + 1(1 + 2t)$$

15. (5.1.6) Yes. -2.

16. (5.1.14) $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

17. (5.1.18) 4, 0, -3.

18. (5.1.19) 6. As it is easy to see $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector corresponding to the eigenvalue 3. (Or 0, as the matrix has rank 1.)

19. (5.1.20) 0. The eigenvectors are $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.

20. (5.1.22)

    (a) F. $\mathbf{x}$ has to be non-zero.

    (b) F. All vectors in $\mathbb{R}^3$ are eigenvector of $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ correspond to eigenvalue 1.

    (c) T. With eigenvalue 1.

(d) F. That rule can be only applied to triangular matrices.

(e) T. $\mathrm{Nul}(A - \lambda I)$.

21. (5.1.27) As $\det B = \det B^T$ for any B, we have

$$\det(A - \lambda I) = \det(A^T - \lambda I)$$

so their roots must be the same.