LECTURE 38

Review of the Semester's Topics

Math 1B is divided into three main parts

I Techniques for Computing Integrals

II Sequences and Series

III Ordinary Differential Equations

It may be of small comfort to know:

No one technique works for all integrals.

No one technique works for all series.

No one technique works for all ODE's.

Today's Lecture will focus on review I and part of II.

<u>Techniques of Integration</u>

- **The Fundamental Theorem of Calculus** can be written these two ways:

$$1: \quad \int_a^b f'(x)dx = f(b) - f(a)$$

$$2: \quad \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

All the techniques of integration are based on the following two rules for *differentiation* :

1. **Chain Rule:**

$$F(g(x))' = f(g(x))g'(x) \quad F' = f$$

2. **Product Rule:**

$$(fg)' = f'g + fg'$$

- **Substitution** :

$$\int f(g(x))g'(x)\,dx = F(g(x)) \qquad \text{where } F' = f$$

$$\int f(u)\,du = F(u)$$

$$u = g(x) \qquad \frac{du}{dx} = g'(x) \quad \text{"}du = g'\,dx\text{"}$$

- **Integration by Parts** : Sometimes this method may be used in non-obvious cases.

$$\int fg'\,dx = fg - \int f'\,gdx$$

$$\int udv = uv - \int vdu \quad \begin{array}{l} u = f(x), v = g(x) \\ du = f'dx, dv = g'dx \end{array}$$

Be sure to check for there vertical asymptotes.

Example 1:

$\int x^{-2}dx = -x^{-1} + C$, but $\int_{-1}^{1} x^{-2}dx$ does not exist.

Sequences and Series

- **Definition**: A **sequence** $\{a_n\}$ is a listing of infinitely many numbers, written in order:

$$a_1, a_2, a_3, \ldots$$

- **Definition**: We say the sequence $\{a_n\}$ has the limit L, written

$$\lim_{n\to\infty} a_n = L$$

provided: for each $\varepsilon > 0$ there exists an integer N so that $|a_n - L| < \varepsilon$ if $n \geq N$.

- **Definition**: A **series** is the sum (**if** it exists) of a sequence:

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \ldots$$

- **Definition**: The n **th partial sum** is

$$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + = \cdots + a_n$$

- **Definition**: We say the **series converges** to s, written

$$\sum_{n=1}^{\infty} a_n = s \text{ provided } \lim_{n\to\infty} s_n = s$$

- **Theorem**. A bounded monotonic sequence converges.

- **If** $\sum_{n=1}^{\infty} a_n$ converges, **then** $\lim_{n\to\infty} a_n = 0$,

  but $\lim_{n\to\infty} a_n = 0 \not\Rightarrow \sum_{n=1}^{\infty} a_n$ converges.

- **Tests for Series:**

  1. If $\lim_{n\to\infty} a_n = 0$, then the series **may** converge. If $\lim_{n\to\infty} a_n \neq 0$, the series **definitely** diverges. This can save you a lot of time.
  2. **Integral Test**: $a_n = f(n)$, $f > 0$ and decreasing.
     (a) If $\int_1^{\infty} f(x)dx$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
     (b) If $\int_1^{\infty} f(x)dx$ diverges, so does $\sum_{n=1}^{\infty} a_n$.

We will continue this review in the next lecture.