Homework, week 5

1. (BC, 178-179) 1(for grade), 2, 3, 6.
2. Suppose $f \to \infty$ as $z \to 0$. Show that f has a pole at $z = 0$.
3. Find the range of the function $e^{-\frac{1}{z}}$ as z varies through deleted disc $0 < |z| < 1$.
4. Describe all singularities of $\frac{1}{\sin(z)}$.
5. (for grade) Describe all singularities of $\frac{\sin(\frac{1}{z})}{(z-1)^2(z^2+1)}$
6. (BC, p. 248) 1; (BC, p. 205-206) 1-6.
7. (for grade) Find singular points of the function $\frac{1}{z^2-z^4}$ and Laurent expansions about these points.
8. Find partial fraction decompositions of

$$\frac{1}{(z-1)^2(z^4-1)} \quad \text{(for grade)} , \quad \frac{z}{(z-1)(z^2+1)}$$

9. Find singularities, their type, and corresponding residues for functions

$$\frac{1}{z^2+z^4}, \quad \frac{e^{1/z}}{1-z}, \quad \frac{1}{\sin z}, \quad sin(1/z)$$