

1. $f = u + iv$ is holomorphic in D . Show that if $u^2 + v^2$ is constant, then f is constant.
2. (for grade) $u = x^2 - y^2 + ax$. Find all v such that $f = u + iv$ is holomorphic for all $z \in \mathbb{C}$.
3. (for grade) $f(z) = e^{-1/z}$ when $z \neq 0$. Show that Cauchy-Riemann equations are satisfied for all $z \in \mathbb{C}$, $z \neq 0$.
4. $f = \sum_{k=0}^{\infty} c_k x^k$ find all c_k such that f is holomorphic.
5. (for grade) Find the radius of convergence of

$$\sum_{n=2}^{\infty} \frac{z^{2n+1}}{(2n+1)!}, \quad \sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$$

6. Find the domain of convergence of

$$\sum_{n=1}^{\infty} n(z-i)^n$$

7. Let f be an entire function (analytical in \mathbb{C}) of the form $f(x, y) = u(x) + iv(y)$.
8. (for grade) B.Ch. p.121, 3
9. B.Ch. 136, 8,9,10.