The character table is almost unitary except that we have to weight the columns by the sizes of the conjugacy classes. The transpose of a unitary matrix is also unitary, so the columns of a character table are orthogonal (for the hermitian inner product) and have norms given by $|G|$/size of conjugacy class which is just the order of the centralizer of an element of the conjugacy class. The orthogonality of characters makes it very easy to work with representations. For example:

- We can count the number of times an irreducible representation occurs in some representation by taking the inner product of their characters.
- A representation is irreducible if and only if its character has norm 1.
- Two representations are isomorphic if and only if they have the same character. (The analogue of this fails in cases when we do not have complete reducibility, such as modular representations of finite groups.)

We can use this to decompose the regular representation: its character is $|G|$ at the identity and 0 elsewhere. So its inner product with the character of any irreducible representation V is $\dim(V)$, so V occurs $\dim(V)$ times. In particular $|G| = \sum \dim(V)^2$, and we can use this to check that a list of irreducibles is complete.

Theorem 267 *The number of irreducible characters of a finite group is equal to the number of conjugacy classes, and the irreducible characters form an orthonormal basis for the class functions.*

Proof The irreducible characters are orthogonal class functions, so it is sufficient to show that the number of conjugacy classes is at most the number of irreducible representations. The key point is to observe that any class function is in the center of the group ring and so by Schur's lemma acts as a scalar on any irreducible representation. If the number of conjugacy classes were greater than the number of irreducibles, we could therefore find a non-zero class function acting as 0 on all irreducibles, and therefore as 0 in the regular representation, which is nonsense. □

It is natural to ask if there is a canonical correspondence between irreducibles and conjugacy classes. At first glance, the answer seems to be obviously no. For example, for infinite compact groups the number of conjugacy classes cane uncountable while the number of irreducibles is countable, and for even the cyclic group of order 3 there is no canonical way to match up the irreducibles with elements of the group. However a close look show that in many cases there does indeed seem to be some sort of natural correspondence. For example, for symmetric groups the conjugacy classes correspond to partitions, and we will later see the same is true for their representations. An even deeper look shows that representations of semisimple Lie groups (and automorphic forms) correspond to conjugacy classes of their "Langlands dual group" though this correspondence need not be 1:1 in general. This is closely related to Langlands functoriality: if the correspondence were 1:1 (which it is not in general) then a homomorphism of Langlands dual groups would induce a map between representations or automorphic forms on different groups. This is expected to hold and is known as Langlands functoriality.

We can describe the irreducible representations of a group most conveniently by giving their character tables: These are just square matrices giving the values of the characters on the conjugacy classes.

One reason why character tables are useful is that they are usually easy to compute. (This only applies to complex character tables: modular character tables are far harder to compute.)

Examples: We compute the character tables of S_3, S_4, S_5 using ad hoc methods. (In fact we will see later how to compute the characters of all symmetric groups in a uniform way.)

We can guess the character table of $SU(2)$. has an obvious 2-dimensional representation. Its conjugacy classes correspond to diagonal matrices with entries u, u^{-1} for u of absolute value 1, except that we can exchange the entries by conjugating by $\left(\begin{smallmatrix} 0 & i \\ i & 0 \end{smallmatrix}\right)$ (generating the Weyl group: note that there is no matrix of order 2 in $SU(2)$ swapping the entries). The trace of this on the 2-dimensional representation is just $u + u^{-1}$. We can also take the representation consisting of polynomials of degree n on this 2-dimensional representation. This has a basis $x^n, x^{n-1}y, \ldots, y^n$ on which the trace is $u^n + u^{n-2} + \cdots u^{-n}$, so this is the character of this $n+1$-dimensional representation. So the characters are given by $(u^{n+1} - u^{-n-1})/(u - u^{-1})$, which will turn out to be a special case of the Weyl character formula. We will soon see that these are all the irreducible representations.

What about orthogonality of characters? The characters are orthogonal even functions on the unit circle if we change the measure by a factor of $(u - u^{-1})^2$. Where has this funny-looking factor come from? The answer is that we should really be integrating over the whole of $SU(2)$, not just over the torus. While the torus does contain a representative of each conjugacy class, we cannot just change integrals of class functions on the group to integrals over the torus, because some conjugacy classes are in some sense bigger than others. The factor $(u - u^{-1})^2$ accounts for the fact that some conjugacy classes are bigger, and is essentially the Weyl integration formula for $SU(2)$.