Math 1B Midterm 2 2011-3-31 2:00-3:30

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simplified form. Write the final answer to each question on the coversheet, and attach the coversheet to your bluebook.

1. Show that $\ln(n) \leq 1/1 + 1/2 + 1/3 + \cdots + 1/n \leq 1 + \ln(n)$.

2. Find the limit of the sequence $(1 + 3/n)^n$.

3. Determine whether the series $\sum_{k=1}^{\infty} \arctan(k) k^{-3/2}$ converges or diverges and give a reason for your answer.

4. Find the values of p for which the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent, and give a reason for your answer.

5. Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n! 10^n}$ converges, and find how many terms are necessary to find the sum with an error less than $\cdot 00001$.

6. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^3 x^n}{(3n)!}$.

7. Use power series to calculate the integral $\int_0^{0 \cdot 1} \ln(x+1) \arctan(x) dx$ to four decimal places.

8. Use power series to evaluate the limit $\lim_{x \to 0} \frac{x^5}{\sin(x) - x + x^3/6}$.