

Math 1B Midterm 2 2009-4-2 12:30-2:00

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simplified form. Write the final answer to each question on the coversheet, and attach the coversheet to your bluebook.

1. Determine whether the sequence $(\ln n)^3/n$ converges or diverges, and give a reason for your answer.
2. Find the limit of the sequence $\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \dots$
3. Determine whether the series $\sum_{k=1}^{\infty} \frac{1+2^k}{3^k-1}$ converges or diverges and give a reason for your answer.
4. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.
5. Find the values of p for which the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent, and give a reason for your answer.
6. Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$ converges, and find how many terms are necessary to find the sum with an error less than .00001.
7. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(-3)^n x^n}{\sqrt[4]{n}}$.
8. Find the first 3 nonzero terms in the Maclaurin series for the function $e^x \ln(1+x)$.
9. Use a power series to calculate the integral $\int_0^{0.1} \frac{x}{1-x^3} dx$ to six decimal places.
10. Find the Taylor series for $\ln(x)$ centered at the point $a = 3$.
11. Use power series to evaluate the limit $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{1 + x - e^x}$.
12. Estimate the range of values for which the approximation $\cos(x) \cong 1 - x^2/2$ is accurate to within an error of $1/240000$.