Math 1B Midterm 1 practice

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simpified form. Write the final answer to each question on the coversheet, and attach the coversheet to your bluebook.

1. Evaluate the indefinite integral $\int x \ln(x)dx$.

2. Evaluate the integral $\int_0^\pi \sin^4(3t)dt$.

3. Evaluate the integral $\int_{\sqrt{2}}^2 \frac{1}{t^3\sqrt{t^2-1}}dt$.

4. Evaluate the integral $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}dy$.

5. Evaluate the integral $\int \frac{x^3}{\sqrt[3]{x^2+1}}dx$.

6. Use the midpoint rule and Simpson's rule with 9 points to approximate the integral $\int_1^5 \frac{\cos(x)}{x}dx$.

7. Use the comparison theorem to determine whether the following integral is convergent: $\int_1^\infty \frac{\cos(x)^2}{1+x^2}dx$.

8. Find the values of p for which the integral $\int_0^1 x^{-p}dx$ converges, and evaluate it for these values of p.

9. Find the length of the curve $y = x^5/6 + 1/10x^3$ for $1 \le x \le 2$.

10. Use Simpson's rule with $n = 10$ to estimate the arc length of the curve $y = \sec(x)$ for $0 \le x \le \pi/3$.

11. If 2J of work is needed to stretch a spring from its natural length of 30cm to 42cm, how much work is needed to stretch it from 35cm to 40cm?

12. Find the centroid of the region bounded by the curves $y = \sin(x)$, $y = \cos(x)$, $x = 0$, $x = \pi/4$.