Math 1B Final practice

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simplified form. Write the final answer to each question on the cover-sheet, and attach the cover-sheet to your bluebook.

1. Use integration by parts to prove $\int \ln(x)^n dx = x \ln(x)^n - n \int \ln(x)^{n-1} dx$.

2. Evaluate the integral $\int \frac{1}{x^3-1} dx$.

3. Find the length of the curve $y = \cosh(x)$ for $0 \le x \le 1$.

4. Solve the differential equation $x \cos(x) = (2y + e^{3y})y'$ with the initial condition $y(0) = 0$.

5. Determine whether the series $\sum_{n=1}^{\infty} ((n^2 + 1)/(2n^2 + 1))^n$ is absolutely convergent, conditionally convergent, or divergent.

6. Use a power series to approximate the integral $\int_0^{0.2} \frac{1}{1+x^5} dx$ to 6 decimal places.

7. For what values of k does the function $y = \sin(kt)$ satisfy the differential equation $y'' + 9y = 0$?.

8. Sketch a direction field for the differential equation $y' = 1 + y$ then use it to sketch 3 solution curves.

9. Use Euler's method with step size 0.1 to estimate $y(0.5)$, where $y(x)$ is the solution of the initial-value problem $y' = y + xy$, $y(0) = 1$.

10. Solve the differential equation $\frac{dy}{dt} = \frac{te^t}{y\sqrt{1+y^2}}$.

11. Find the orthogonal trajectories of the family of curves $y = 1/(x + k)$.

12. The half life of cesium-137 is 30 years. Suppose we have a 100mg sample. Find the mass that remains after t years. After how long will only 1mg remain?

13. For the differential equation $\frac{dy}{dt} = ky(1 - ky)$ find $y(1)$ given $y(0) = 2 \times 10^7$, $K = 8 \times 10^7$, $k = .71$.

14. Solve the differential equation $xy' + y = \sqrt{x}$.

15. Solve the initial value problem $y' = x + y$, $y(0) = 2$.

16. The Lokta-Volterra equations $\frac{dr}{dt} = 0.08r(1 - .0002r) - .001rw$, $\frac{dw}{dt} = -.02w + .00002rw$ models populations of wolves and rabbits. Find all equilibrium solutions and explain their significance.

17. Solve the differential equation $y'' + y' + y = 0$.

18. Solve the boundary value problem $y'' - 6y' + 25y = 0$, $y(0) = 1$, $y(\pi) = 2$.

19. Solve the differential equation $y'' + y = e^x + x^3$ using the method of undetermined coefficients.

20. Solve the differential equation $y'' - y = 1/x$ using the method of variation of parameters.

21. A spring with a 3-kg mass is held stretched 0.6m beyond its natural length by a force of 20N. If the spring begins at equilibrium position but a push gives it an initial velocity of 1.2m/s, find the position of the mass after t seconds.

22. Use power series to solve the differential equation $y' = x^2 y$.