Math 1B Final 2011-5-9 11:30am-2:30pm

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simplified form. Write the final answer to each question on the cover-sheet, and attach the cover-sheet to your bluebook.

1. The Fibonacci numbers satisfy $f(0) = 0$, $f(1) = 1$, and $f(n) = f(n-1) + f(n-2)$. Find an explicit formula for $f(n)$ of the form $f(n) = ab^n + cd^n$ for constants a, b, c, d that you should determine.

2. Sketch a direction field for the differential equation $y' = 1 - xy$ then use it to sketch the solution passing though (0,0).

3. Use Euler's method with step size 1 to estimate $y(3)$, where $y(x)$ is the solution of the initial-value problem $y' = 1 - xy$, $y(0) = 0$.

4. Solve the separable differential equation $\frac{dy}{dx} = y^2 \sin(x)$.

5. Find the orthogonal trajectories of the family of curves $y = ke^x$.

6. Find all solutions of the differential equation $x^2 y' + 2xy = \cos x$ for $x > 0$.

7. Solve the Bernoulli differential equation $y' + 2y/x = y^3/x^2$ by using the substitution $u = 1/y^2$ to convert it into a linear equation, and then solving this linear equation.

8. Find all solutions of the differential equation $y'' - 8y' + 12y = 0$.

9. Either solve the following boundary value problem or show that it has no solutions: $4y'' + y = 0$, $y(0) = 2$, $y(\pi) = 3$.

10. Solve the initial value problem $y'' - y = 2e^x$, $y(0) = 1$, $y'(0) = 0$ using the method of undetermined coefficients.

11. Solve the differential equation $y'' + 3y' + 2y = \sin(e^x)$ using the method of variation of parameters. (Write $y = u_1 y_1 + u_2 y_2$ where y_1 and y_2 are the solutions of the homogeneous equation, then solve for u_1 and u_2 satisfying the extra condition $u_1' y_1 + u_2' y_2 = 0$.)

12. Find the first 3 non-zero terms of the power series of the function y satisfying the differential equation $x^2 y'' + xy' + x^2 y = 0$ and $y(0) = 1$.