Math 1B Final 2009-5-19 5:00-8:00pm

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simplified form. Write the final answer to each question on the cover-sheet, and attach the cover-sheet to your bluebook.

1. Evaluate the integral $\int_1^2 (\ln x)^2 dx$.
2. Find the length of the curve $y = \cosh(x)$ for $-1 \le x \le 1$.
3. Find the first 3 non-zero terms of the Maclaurin series for the function $e^x \ln(1-x)$.
4. Sketch a direction field for the differential equation $y' = x + xy$ then use it to sketch the solution passing though (0,-1).
5. Use Euler's method with step size 1 to estimate $y(2)$, where $y(x)$ is the solution of the initial-value problem $y' = y + xy$, $y(0) = 1$.
6. Solve the separable differential equation $\frac{dy}{dx} = \frac{e^{2x}}{y^3}$.
7. Find the orthogonal trajectories of the family of curves $x^2 - y^2 = k$.
8. Solve the logistic differential equation $\frac{dy}{dt} = y(1-y)$ with initial condition $y(0) = 1/2$.
9. Solve the linear differential equation $xy' - 2y = 2x^2$.
10. Solve the Bernoulli differential equation $xy' + y = -xy^2$ by using the sustitution $u = 1/y$ to convert it into a linear equation, and then solving this linear equation.
11. Solve the differential equation $y'' - y' + y = 0$.
12. Solve the initial value problem $y'' - 2y' + y = 0$, $y(0) = 1$, $y'(0) = 0$.
13. Either solve the following boundary value problem or show that it has no solutions: $y'' + 100y = 0$, $y(0) = 2$, $y(\pi) = 3$.
14. Solve the initial value problem $y'' - y = xe^x$, $y(0) = 2$, $y'(0) = 1$ using the method of undetermined coefficients.
15. Solve the differential equation $y'' + y = 1/\sin(x)$ using the method of variation of parameters. (Write $y = u_1y_1 + u_2y_2$ where y_1 and y_2 are the solutions of the homogeneous equation, then solve for u_1 and u_2 satisfying the extra condition $u_1'y_1 + u_2'y_2 = 0$.)
16. Use power series to solve the differential equation $y' = xy$.