

Math 1A Midterm 1 2006-9-28 2:00-3:30pm.

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simplified form.

1. Find the domain of the function $g(x) = \frac{1}{\sqrt{x^2-6x}}$.

2. Sketch the graph of $y = x \sin(x)$ for $-2\pi \leq x \leq 2\pi$.

3. Sketch the graph of the function $f(x) = x^3 + 1$. Find a formula for its inverse f^{-1} and sketch the graph of f^{-1} on the same plot.

4. Determine the infinite limit

$$\lim_{x \rightarrow 0} \frac{x-1}{x^4(x+3)}$$

5. Evaluate the limit

$$\lim_{x \rightarrow 2} \frac{x^2-4}{x^3-8}$$

6. If $f(x) = x^2$, find a number δ so that $|f(x) - 1| < 1/2$ whenever $|x - 1| < \delta$.

7. Find the numbers at which f is discontinuous, where f is defined by $f(x) = x + 1$ if $x \leq 1$, $f(x) = 1/x$ if $1 < x < 3$, $f(x) = \sqrt{x-3}$ if $x \geq 3$.

8. What is

$$\lim_{x \rightarrow +\infty} \sqrt{\frac{12x^3 - 5x + 2}{1 + 4x^2 + 3x^3}}$$

9. Find the equation of the tangent line to the curve $y = x^4 - 1$ at the point where $x = 1$.

10. State the definition of the derivative of a function, and find the derivative of the function $f(x) = x^3$ using the definition of the derivative.

11. Sketch the graph of a function for which $f(0) = 0$, $f'(0) = -1$, $f(1) = 0$, $f'(1) = -1$.

12. Differentiate the function $y = e^{x+2} + 4\pi^2 + (x^2 + 1)/\sqrt{x}$.

13. At what point on the curve $y = 2 + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 1$?

14. Differentiate $x^2 e^x (\sqrt{x} - 1)$.

15. Differentiate

$$\frac{\sqrt{x} + 1}{\sqrt{x} - 1}$$