Math 1A Midterm 1. 2004-9-30 2:00pm-3:30pm

You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 1 mark, which will be given only for a clear correct answer and correct working. There is no partial credit for wrong answers. There are questions on both sides of the paper.

1. Find the domain of the function $f(x) = \sqrt{x-4}$.

2. Sketch the graph of $y = |x^2 - 1|$.

3. Find a formula for the inverse of the function $y = e^{\sqrt{x}}$.

4. Determine the infinite limit ($+\infty$ or $-\infty$) of
$$\lim_{x \to 0} \frac{x+1}{x^2(x-1)}.$$

5. Evaluate the limit
$$\lim_{x \to -1} \frac{x^2 - x - 2}{x+1}.$$

6. Find the constant c that makes g continuous for all reals, where $g(x) = x^2 - c^2$ if $x < 4$, $g(x) = cx + 20$ if $x \geq 4$.

7. Find the numbers at which f is discontinuous, where f is defined by $f(x) = x + 1$ if $x < 0$, $f(x) = e^x$ if $0 \leq x \leq 1$, $f(x) = 2 - x$ if $x > 1$.

8. Evaluate
$$\lim_{x \to +\infty} \frac{4x^2 - 3}{x^2 - x + 10}.$$

9. Find the equation of the tangent line to the curve $y = x^3$ at the point where $x = 1$.

10. Sketch the graph of a function for which $f(0) = 0$, $f'(0) = -1$, $f(1) = 0$, $f'(1) = -1$.

11. Determine for what values of x the function $f(x) = x|x|$ is differentiable and find a formula for f'.

12. Differentiate the function $y = 6x^{-8/3}$.

13. Find all points on the curve $y = x^3 + 3x^2 + 3x + 1$ where the tangent is horizontal.

14. Differentiate $(x^3 + 1)e^x$.

15. Differentiate
$$\frac{e^x - 1}{e^x + 1}.$$

Answers:

1. $x \geq 4$

2. Graph should come down to $(-1, 0)$, then go back up and come down again at $(1, 0)$, then go up again. At these 2 points the graph should have "corners".

3. $x = \log(y)^2$

4. $-\infty$

5. -3

6. $c = -2$

7. $x = 1$

8. 4

9. $y - 1 = 3(x - 1)$ or $y = 3x - 2$

10. Graph of function should pass though $(0, 0)$ and $(1, 0)$ and should slope down at both these points.

11. All x, derivative is $2|x|$.

12. $-16x^{-11/3}$.

13. $(x, y) = (-1, 0)$. (Accept just $x = -1$ as correct.)

14. $(x^3 + 3x^2 + 1)e^x$.

15. $2e^x/(e^x + 1)^2$