

Math 1A Final 2012 Dec 14 7:00pm-10:00pm

Name	Student ID	Name of GSI
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You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simplified form. Write the final answer on this cover-sheet. There are questions on both sides of the paper.

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1. Evaluate the limit $\lim_{t \rightarrow 0} \frac{\sqrt{2+t} - \sqrt{2-t}}{t}$.

2. Find dy/dx if $x \sin(y) + y \sin(x) = 2$.

3. If $f(1) = 2$ and $f'(x) \leq 3$ for all x , what is the largest value that $f(5)$ could be?

4. Find $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - x^2/2}{x^3}$.

5. Use two steps of Newton's method starting with the initial approximation $x = 1$ to estimate a root of the equation $x^3 - 2x - 1 = 0$.

6. Find f given that $f''(t) = 2e^t + 3 \sin(t)$, $f(0) = 0$, $f(\pi) = 0$.

7. Evaluate the integral $\int_{-2}^0 (1 + \sqrt{4 - x^2}) dx$ by interpreting it as an area.

8. Find the derivative of the function $g(x) = \int_x^\pi \sqrt{1 + \sec(t)} dt$.

9. Find the indefinite integral $\int \frac{1}{x^2(x+2)} dx$ by writing the integrand in the form $a/x^2 + b/x + c/(x+2)$.

10. Evaluate the integral $\int_{\pi/4}^{\pi/3} \frac{1}{(\sin x)^2} dx$.

11. Find the area of the finite region bounded by the lines $x = 0$, $x = 2y - y^2$.

12. Estimate $1 + 1/2 + 1/3 + \cdots + 1/1000000000$ as a decimal number with an error of less than $1/2$, given that $\ln(10)$ is about 2.30.

13. Evaluate the indefinite integral $\int \frac{\ln(x)}{x} dx$.

14. Evaluate the definite integral $\int_0^2 xe^x dx$.

15. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$, $x = 1$, about the y -axis.

16. Find the volume of the region obtained by rotating the region bounded by the curves $y = x^4$, $y = 0$, $x = 1$, about the x -axis.