

Math 185 midterm 1, 2009 Sept 29 3:30-5:00.

Please make sure that your name is on everything you hand in. You are allowed 1 sheet of notes. Calculators are not allowed. Each question is worth 3 marks, which will only be given for correct working and a clear and correct answer in simplified form.

1. Find all values of $(1 + i\sqrt{3})^{20}$ and i^i .

2. Evaluate the contour integral

$$\int_C z^n dz$$

where n is an integer (possibly negative) and the contour C is given by $C(t) = 2e^{2\pi it}$ for $0 \leq t \leq 1$.

3. Show that $u(x, y) = \cosh(x) \cos(y)$ is a harmonic function of x and y . Find a harmonic conjugate v of u and an analytic function whose real part is u .

4. Find where each of the following functions are analytic: $f(z) = (z^3 - i)/(z^2 - 4z + 3)$, $g(x + iy) = e^y e^{ix}$, $h(x + iy) = \sin(x) \cosh(y) + i \cos(x) \sinh(y)$.

5. Express $\cos(5\theta)$ and $\sin(5\theta)$ as polynomials in $\cos(\theta)$ and $\sin(\theta)$.

6. Find and sketch the images of the hyperbolas $x^2 - y^2 = a$ and $xy = b$ under the transformation $w = z^2$, where a and b are real constants and $z = x + iy$.

Solutions.

1. Use polar coordinates: $2^{20} e^{2\pi i/3}$ and $e^{2n\pi - \pi/2}$.

2. 0 unless $n = -1$ in which case it is $2\pi i$.

3. $v = \sinh(x) \sin(y)$, $u + iv$ is analytic.

4. Everywhere except 1 and 3. Nowhere. Everywhere. (For the last 2 check the CR equations)

5. Use $\cos(5\theta) + i \sin(5\theta) = (\cos(\theta) + i \sin(\theta))^5 = \cos(\theta)^5 + 5i \cos(\theta)^4 \sin(\theta) - 10 \cos(\theta)^3 \sin(\theta)^2 - 10i \cos(\theta)^2 \sin(\theta)^3 + 5 \cos(\theta) \sin(\theta)^4 + i \sin(\theta)^5$ and equate real and imaginary parts of both sides.

6. The images are vertical and horizontal lines.