Math 121B midterm 2013 April 4 11:00-12:30.

Please make sure that your name is on everything you hand in.

You are allowed calculators and 1 page of notes. Full credit will only be given for correct working and a clear and correct answer in simplified form.

All questions have about the same number of marks.

1. Solve the following differential equation by the method of Frobenius (generalized power series) finding the first three nonzero coeffcients of the solutions:

$$2xy'' - y' + 2y = 0.$$

2. Express $J_{1/2}(x)$ in terms of $\sin x$, using the definition

$$J_p(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x/2)^{2n+p}}{n!\Gamma(n+p+1)}.$$

3. Define the Hermite polynomial $H_n(x)$ to be $(-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$. Use this definition to prove that H_n is a polynomial of degree n, and that if $m \neq n$ then $\int_{-\infty}^{\infty} H_m(x) H_n(x) e^{-x^2} dx = 0$.

4. The Laguerre polynomial is $L_n(x) = \frac{1}{n!} e^x \frac{d^n}{dx^n}(x^n e^{-x})$. Use this to find the Laguerre polynomial $L_4(x)$.

5. Find the displacement $u(x,t)$ of a string whose ends $-\pi/2$ and $\pi/2$ are fixed, if at time $t = 0$ the displacement is $u(x,t) = \pi/2 - |x|$ and the initial velocity is $\frac{\partial}{\partial t} u(x,t) = 0$. The displacement u satisfies the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}.$$