Math 121B practice midterm.

Please make sure that your name is on everything you hand in.
You are allowed calculators and 1 page of notes.
All questions have about the same number of marks.

1. Solve the following differential equation by the method of Frobenius (generalized power series):

$$x^2y'' - 6y = 0.$$

2. Express $\frac{d}{dx}J_0(x)$ in terms of $J_1(x)$, using the definition

$$J_p(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x/2)^{2n+p}}{n!(n+p)!}.$$

3. Use the relation

$$\exp(2xh - h^2) = \sum_{n=0}^{\infty} \frac{H_n(x)h^n}{n!}$$

to calculate the Hermite polynomials H_0, H_1, H_2, and H_3. What is the coefficient of x^n of $H_n(x)$?

4. The Laguerre differential equation is

$$xy'' + (1-x)y' + py = 0.$$

Find the polynomial solution $L_p(x)$ with constant term 1 for $p = 3$.

5. A bar of length π with insulated sides is initially at a temperature of 1. Starting at time $t = 0$, the ends are held at a temperature of 0. Find the temperature distribution $T(x,t)$ in the bar at time t. The temperature T satisfies the heat equation

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}.$$