**Math 121B first midterm 2013-2-21 11:00-12:30.**

Please make sure that your name is on everything you hand in. You are allowed calculators and 1 page of notes. All questions have about the same number of marks. Full credit will only be given for correct working and a clear and correct answer in simplified form.

1. Express the integral
$$\int_0^\infty y^6 e^{-y^2}\,dy$$
as a gamma function, and use this to evaluate it explicitly.

2. Use Stirling's formula $\Gamma(n) \cong n^{n-1}e^{-n}\sqrt{2\pi n}$ to evaluate
$$\lim_{n\to\infty} \frac{\Gamma(n+3/2)}{\Gamma(n+1)\sqrt{n}}.$$

3. Find the general solution of
$$y'' - 4xy' + (4x^2 - 2)y = 0$$
by writing $y$ as a power series $a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$ in $x$.

4. Find the first five coefficients $a_0, a_1, a_2, a_3, a_4$ of the expansion of $f(x) = \sum_n a_n P_n(x)$ as a series of Legendre polynomials, where $f(x) = -1$ for $-1 < x < 0$ and $f(x) = 1$ for $0 < x < 1$. Find the best (in the least squares sense) third degree polynomial approximation to the function $f$. (The first few Legendre polynomials are $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2 - 1)/2$, $P_3(x) = (5x^3 - 3x)/2$, $P_4(x) = (35x^4 - 30x^2 + 3)/8$.)

5. Find the coefficient of $x^3$ in the Legendre polynomial $P_5(x)$ using either Rodrigues' formula
$$P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2 - 1)^l$$
or any other method that would work in general. (You will get no credit just for copying $P_5$ from your notes. To get full credit for this question you have to find the coefficient using a method that would work for any other Legendre polynomial.)