

**Math 121B midterm, Thursday 2004 Feb 19, 9:30-11:00am.**

Please make sure that your name is on everything you hand in.

You are allowed calculators and 1 page of notes.

All questions have about the same number of marks.

1. Express the integral

$$\int_0^{\infty} \frac{y^2 dy}{(1+y)^6}$$

as a beta function, hence in terms of gamma functions, and use this to evaluate it explicitly. (Hint: put  $x = y/(1+y)$  in the definition  $B(p, q) = \int_0^1 x^{p-1}(1-x)^{q-1} dx = \Gamma(p)\Gamma(q)/\Gamma(p+q)$ .)

2. Use Stirling's formula  $n! \cong n^n e^{-n} \sqrt{2\pi n}$  to evaluate

$$\lim_{n \rightarrow \infty} \frac{(2n)! \sqrt{n}}{2^{2n} (n!)^2}.$$

3. Find the general solution of

$$(x^2 + 1)y'' - 2xy' + 2y = 0$$

by writing  $y$  as a power series  $a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$  in  $x$ .

4. Find the best (in the least squares sense) second-degree polynomial approximation  $a_0 + a_1x + a_2x^2$  to the function  $x^4$  for  $-1 \leq x \leq 1$ . (The first few Legendre polynomials are  $P_0(x) = 1$ ,  $P_1(x) = x$ ,  $P_2(x) = (3x^2 - 1)/2$ ,  $P_3(x) = (5x^3 - 3x)/2$ ,  $P_4(x) = (35x^4 - 30x^2 + 3)/8$ .)

5. Find  $P_6(0)$  from Rodrigues' formula

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l.$$