Math 121B practice midterm 1.

Please make sure that your name is on everything you hand in. You are allowed calculators and 1 page of notes. All questions have about the same number of marks.

1. Express the integral

$$\int_0^\infty \frac{y^2 dy}{(1+y)^6}$$

as a beta function, hence in terms of gamma functions, and use this to evaluate it explicitly. (Hint: put x = y/(1+y) in the definition $B(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1} dx = \Gamma(p)\Gamma(q)/\Gamma(p+q).$)

2. Use Stirling's formula $n! \cong n^n e^{-n} \sqrt{2\pi n}$ to evaluate

$$\lim_{n \to \infty} \frac{(2n)!\sqrt{n}}{2^{2n}(n!)^2}.$$

3. Find the general solution of

$$(x^2 + 1)y'' - 2xy' + 2y = 0$$

by writing y as a power series $a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$ in x.

- 4. Find the best (in the least squares sense) second-degree polynomial approximation $a_0 + a_1x + a_2x^2$ to the function x^4 for $-1 \le x \le 1$. (The first few Legendre polynomials are $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2 1)/2$, $P_3(x) = (5x^3 3x)/2$, $P_4(x) = (35x^4 30x^2 + 3)/8$.)
- 5. Find $P_6(0)$ from Rodrigues' formula

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l.$$