## Math 121B practice midterm 1.

Please make sure that your name is on everything you hand in.
You are allowed calculators and 1 page of notes.
All questions have about the same number of marks.

1. Express the integral

$$
\int_{0}^{\infty} \frac{y^{2} d y}{(1+y)^{6}}
$$

as a beta function, hence in terms of gamma functions, and use this to evaluate it explicitly. (Hint: put $x=y /(1+y)$ in the definition $\left.B(p, q)=\int_{0}^{1} x^{p-1}(1-x)^{q-1} d x=\Gamma(p) \Gamma(q) / \Gamma(p+q).\right)$
2. Use Stirling's formula $n!\cong n^{n} e^{-n} \sqrt{2 \pi n}$ to evaluate

$$
\lim _{n \rightarrow \infty} \frac{(2 n)!\sqrt{n}}{2^{2 n}(n!)^{2}}
$$

3. Find the general solution of

$$
\left(x^{2}+1\right) y^{\prime \prime}-2 x y^{\prime}+2 y=0
$$

by writing $y$ as a power series $a_{0}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}+\cdots$ in $x$.
4. Find the best (in the least squares sense) second-degree polynomial approximation $a_{0}+a_{1} x+a_{2} x^{2}$ to the function $x^{4}$ for $-1 \leq x \leq 1$. (The first few Legendre polynomials are $P_{0}(x)=1, P_{1}(x)=x, P_{2}(x)=$ $\left.\left(3 x^{2}-1\right) / 2, P_{3}(x)=\left(5 x^{3}-3 x\right) / 2, P_{4}(x)=\left(35 x^{4}-30 x^{2}+3\right) / 8.\right)$
5. Find $P_{6}(0)$ from Rodrigues' formula

$$
P_{l}(x)=\frac{1}{2^{l} l!} \frac{d^{l}}{d x^{l}}\left(x^{2}-1\right)^{l} .
$$

