Math 121B final 2013 May 16 8:00-11:00

Please make sure that your name is on everything you hand in. You are allowed calculators and 1 page of notes. Full credit will only be given for correct working and a clear and correct answer in simplified form. All questions have about the same number of marks.

1. Find the general solution of the differential equation $x^2y'' = y$.

2. By repeated integration by parts, find the first 3 terms of the asymptotic series for the incomplete gamma function $\int_x^{\infty} t^{n-1} e^{-t} dt$.

3. One out of every 10000 people has a certain rare disease without realizing it. A test for the disease gives the correct answer 99 percent of the time (whether or not you have the disease). If you are tested for the disease and the test is positive, estimate roughly the chance that you have the disease. If you are tested twice and both tests are positive estimate roughly the chance you have the disease (assuming the tests are independent).

4. The fairground dice game Chuck-a-luck involves throwing three 6-sided dice. You win a dollar for each 6 that appears, unless no 6's appear in which case you lose a dollar. Calculate your expected loss.

5. Five people have their hats blown off by a gust of wind, and each randomly picks up one of the hats. Calculate the probability that no-one has their own hat.

6. Two cards are drawn from a shuffled deck of 10 cards with 5 red and 5 black cards. What is the probability that both are red? What is the probability that both are red given that at least one is red? What is the probability that both are red given that the first one is red?

7. A random variable is given by the sum of the numbers on two 6-sided dice. Find its mean and variance.

8. If a new car has on average 3 (independent) defects, use the Poisson distribution to estimate the probability that a new car you buy at has most 1 defect.