

Math 121 midterm, Tuesday October 30, 9:40-11:00.

Please make sure that your name is on everything you hand in.

You are allowed calculators and 1 page of notes.

All questions have about the same number of marks.

1. Find the point on the sphere $x^2 + y^2 + z^2 = 1$ for which $x + 2y + 2z$ is a maximum.
2. If $w = \int_x^{x^2} e^{-u^2} du$ find dw/dx .
3. Show that $x/(x^2 + y^2)$ is harmonic (in other words, it satisfies Laplace's equation). Find an analytic function $f(z) = f(x + iy)$ of which it is the real part. (Hint: the function has degree -1 , so the same is likely to be true for f .) Find the conjugate harmonic function $Im(f(x + iy))$.
4. Evaluate the contour integral $\int_C e^z dz / (z - 2)$ if C is the circle of center 0 and radius 3.
5. Find the Laurent series for $\sin(\pi z) / (4z^2 - 1)$ about the point $z = 1/2$ and use this to find the residue at $z = 1/2$.
6. Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{\cos(x)}{x^2 + 1} dx$$

(Hint: $\cos(x) = Re(\exp(ix))$.)

7. Evaluate the integral

$$\int_0^{2\pi} \frac{d\theta}{5 - 3 \cos(\theta)}$$

(Hint: put $z = e^{i\theta}$.)

8. A flat plate is in the shape of a quarter circle of radius 1, and consists of the points (x, y) in the plane with $x \geq 0$, $y \geq 0$, $x^2 + y^2 \leq 1$. The curved part of the boundary is insulated and the edges $y = 0$ and $x = 0$ are held at temperatures of 0° and 100° . Find the temperature distribution $T(x, y)$ inside the plate. (Hint: if the quarter circle is regarded as part of the unit circle in the z -plane, then the mapping function $w = \log(z)$ maps the quarter circle to an infinitely long rectangle in the w -plane.)