Homework .

5.1 $dx/dt = \sinh(t)$, $dy/dt = -\sin(t)$, $dz/dt = (\partial z/\partial x)dx/dt + (\partial z/\partial y)dy/dt = e^{-y}\sinh(t) + (-xe^{-y})(-\sin(t))$

5.4 $dx/dt = (2u/(u^2-v^2)) \times 2t + (-2v/(u^2-v^2)) \times (-\sin(t))$.

6.1 Differential with respect to p: $v^a + pav^{a-1}(dv/dp) = 0$. So $dv/dp = -v/ap$. Differentiating $v + ap(dv/dp) = 0$ again with respect to p gives $dv/dp + adv/dp + ap(d^2v/dp^2) = 0$. So $d^2v/dp^2 = -(1+a)(-v/ap)/ap$

6.2 $y'e^{xy} + y^2e^{xy} + yxy'e^{xy} = \cos(x)$, so $y' = 1$ at $x = y = 0$. This gives $e^{xy}(y' + y^2 + yxy') = \cos(x)$, and differentiating this gives $(y + xy')e^{xy}(y' + y^2 + yxy') + e^{xy}(y'' + 2yy' + yy' + xy'^2 + yxy'') = -\sin(x)$ For $x = y = 0, y' = 1$ this gives $y'' = 0$.

6.3 Taking differential we have $yx^{y-1}dx + \log(x)x^y dy = xy^{x-1}dy + \log(y)y^x dx$, which for $x = 2$, $y = 4$ gives $32dx + 16\log(2)dy = 8dy + 16\log(4)dx$ so $dy/dx = (16\log(2) - 8)/32(\log(2) - 1)$.

6.5 $y^3dx + 3xy^2dy - x^3dy - 3x^2ydx = 0$, so at $(1, 2)$ we have $dy/dx = -2/11$. So the equation is $2x+11y = 24$.

6.6 We have $y^3 + 3xy^2y' - x^3y' - 3x^2y = 0$. Differentiating again gives $3y^2y' + 3y^2y' + 6xy(y')^2 + 3xy^2y'' - 3x^2y' - x^3y'' - 6xy - 3x^2y' = 0$. For $x = 1, y = 2, y' = -2/11$ this gives $y'' = 1800/11^3$.

7.1 $dx = ydz + zdy$ and $dy = 2\cos(y + z)(dy + dz)$. Eliminating dz gives $dy = 2\cos(y + z)(dy + (dx - zdy)/y)$. So $dx/dy = y(1/2\cos(y + z) - 1 + z/y) = \tan(y + z) - y + z$. Differentiating again gives $d^2x/dy^2 = \sec^2(y + z)(1 + dz/dy) - 1 + (dz/dy)$. Since $dz/dy = 1/2\cos(y + z) - 1$ this gives $d^2x/dy^2 = \sec^3(y + z)/2 + \sec(y + z)/2 - 2$.

7.2 $dP/dt = \cos(t)dr/dt - r\sin(t)$ from the first equation, and the second gives $dr\sin(t) + r\cos(t)dt - 2e^r dt - 2te^r dr = 0$ so $dr/dt = (2e^r - r\cos(t))/(\sin(t) - 2te^e)$ and hence $dP/dt = \cos(t)(2e^r - r\cos(t))/(\sin(t) - 2te^e) - r\sin(t)$.

7.4 $dw = (-2rdr - 2sds)w$, $dr = udv + vdu$, $ds = du + 2dv$, so $dw = (-2r(udv + vdu) - 2s(du + 2dv))w$. So $\partial w/\partial u = (-2rv - 2s)w$ and $\partial w/\partial v = (-2ru - 4s)w$

7.5 $du = 2xy^3zdx + 3x^2y^2zdy + x^2y^3dz$, $dx = \cos(s+t)(ds+dt)$, $dy = -\sin(s+t(ds+dt)$, $dz = (sdt+tds)z$. So $du = 2xy^3z\cos(s+t)(ds+dt) + 3x^2y^2z(-\sin(s+t)(ds+dt)) + x^2y^3(sdt+tds)z$, which implies $\partial u/\partial s = 2xy^3z\cos(s+t) + 3x^2y^2z(-\sin(s+t)) + x^2y^3(t)z$ $\partial u/\partial t = 2xy^3z\cos(s+t) + 3x^2y^2z(-\sin(s+t)) + x^2y^3sz$.

7.8 $s^2dx + 2xsds + t^2dy + 2ytdt = 0$ and $2xsdx + x^2ds + 2ytdy + y^2dt = xdy - ydx$. For $(x, y, s, t) = (1, -3, 2, -1)$ this gives $4dx + 4ds + dy + 6dt = 0$ and $4dx + ds + 6dy + 9dt = dy - 3dx$, so $7dx + ds + 5dy + 9dt = 0$. Eliminating dy gives $-13dx - 19ds - 21dt = 0$ so $\partial x/\partial s = -19/13$, $\partial x/\partial t = -21/13$. Eliminating dx gives $24ds - 13dy + 6dt = 0$ so $\partial y/\partial s = 24/13$, $\partial y/\partial t = 6/13$.

7.10 $2xdx + 2ydy = 2sdt + 2tds$ and $2xdy + 2ydx = 2sds - 2tdt$. For $(x, y, s, t) = (4, 2, 5, 3)$ this gives $8dx + 4dy = 10dt + 6ds$ and $8dy + 4dx = 10ds - 6dt$. Eliminating dy gives $-12dx = -2ds - 26dt$ so $\partial x/\partial s = 2/12 = 1/6$, $\partial x/\partial t = 26/12 = 13/6$. Eliminating dx gives $-12dy = 22dt - 14ds$ so $\partial y/\partial s = 14/12 = 7/6$, $\partial y/\partial t = -22/12 = -11/6$.

7.15 $2xudx + x^2du - 2yvdy - y^2dv = 0$ and $dx + dy = udv + vdu$. If we put $dv = 0$ and eliminate dy we find $2xudx + x^2du - 2yv(vdu - dx) = 0$ so $\partial x/\partial u)_v = -(x^2 - 2yv^2)/(2xu + 2yv)$. If we put $dy = 0$ and eliminate dv we find $2xudx + x^2du - y^2(dx - vdu)/u = 0$, so that $(\partial x/\partial u)_y = -(x^2 + y^2v/u)/(2xu - y^2/u) = (x^2u + y^2v)/(y^2 - 2xu^2)$

7.19 $dz = dr + 2sds$, $dx + dy = 3s^2ds + 3r^2dr$, $xdy + ydx = 2sds - 2rdr$, so we have $dz = dr + 4ds$, $dx + dy = 12ds + 3dr$, $3dy + dx = 4ds + 2dr$ at $(r, s, x, y, z) = (-1, 2, 3, 1, 3)$. For $(\partial x/\partial z)_s$ we put $ds = 0$ and eliminate dr, dy from the equations using $dr = dz$, $dy = 3dz - dx$ to find that $9dz - 3dx + dx = 2dz$, so $(\partial x/\partial z)_s = 7/2$. For $(\partial x/\partial z)_r$ we put $dr = 0$ and eliminate ds, dy from the equations using $ds = dz/4$, $dy = 12ds - dx$ to find that $9dz - 3dx + dx = dz$, so $(\partial x/\partial z)_r = 4$. For $(\partial x/\partial z)_y$ we put $dy = 0$ and eliminate ds, dr from the equations to find $dx = 3dz$, $9dz - 3dx + dx = dz$, so $(\partial x/\partial z)_y = 3$.