Homework 7.

1.2 $\partial s/\partial t = t^{u-1}$, $\partial s/\partial u = t^u \log(t)$.

1.4 $\partial w/\partial x = 3x^2 - 2y$ $\partial w/\partial y = -3y^2 - 2x$. The only real solutions to both of these being zero are $(x, y) = (0, 0)$ or $(-2/3, 2/3)$. (There are also a couple of complex solutions.) At $(0, 0)$, $\partial^2 w/\partial x^2 = \partial^2 w/\partial y^2 = 0$. . At $(-2/3, 2/3)$, $\partial^2 w/\partial x^2 = 6x = -4$, $\partial^2 w/\partial y^2 = -6y = -4$.

1.6a $\partial^2 u/\partial x \partial y = -e^x \sin(y) = \partial^2 u/\partial y \partial x$.

1.6b $\partial^2 u/\partial x^2 = e^x \cos(y) = -\partial^2 u/\partial y^2$.

1.7 $2x$

1.8 $z = 2r^2 - x^2$ so $(\partial z/\partial x)_r = -2x$.

1.9 $z = x^2(1 + 2\tan^2(\theta)$, so $(\partial z/\partial x)_\theta = 2x(1 + 2\tan^2(\theta))$.

1.23 $z = r^2(1 + \sin(\theta)^2)$, so $\partial^2 z/\partial r \partial \theta = 4r \sin(\theta)\cos(\theta) = 2r \sin(2\theta)$

1.24 $\partial^2 z/\partial x \partial y = 0$.

2.3 Multiply series for $\log(1 + x)$ by series for $1/(1 + y)$ to get $x - x^2/2 - xy + x^3/3 + x^2y/2 + xy^2 + \cdots$

2.6 Substitute $x + y$ into series for exp to get $1 + x + y + x^2/2 + xy + y^2/2 + \cdots$.

2.8 $e^x \cos(y) = 1 + x + x^2/2 - y^2/2 + x^3/6 - xy^2/2 + \cdots$, $e^x \sin(y) = y + xy + x^2y/2 - y^3/6 + \cdots$

4.1 The difference is about $d/dn(n^{-3}) = -3n^{-4}$.

4.2 The difference is about $a\frac{d}{dn}(\sqrt{n}) = a/2\sqrt{n}$. For $a = 5, n = 10^{12}$ this is about 2.5×10^{-6}.

4.10 Component $= \cos(\theta) \times f$ (f=force, θ= angle) so $d(component) = \cos(\theta)df - \sin(\theta)f d\theta$. For $f = 500$, $df = \pm 1$, $\theta = 60^o = \pi/3$, $d\theta = \pm .5^o = .5\pi/180$, we find that the error in the component is at most about $\cos(\pi/3) \times 1 + \sin(\pi/3) \times 500 \times .5\pi/180$ which is about $4nt$. (The book gives 4.28nt; this is excessive precision as the initial values are only give to one significant figure so the final answer should NOT be given to 3 significant figures!)

4.11 $d(\sqrt{x^2 - y^2}) = (2xdx - 2ydy)/2\sqrt{x^2 - y^2})$. For $x = 5, dx = -.02, y = 3, dy = .03$ this gives a change of about $(2 \times 5 \times (-.02) - 2 \times 3 \times .03)/2\sqrt{5^2 - 3^2} = -.0475$. So the answer is about $\sqrt{5^2 - 3^2} - .0475$ which is about 3.9525. (For comparison, the exact answer is 3.95215 . . .)

4.13 $d(\sqrt{x^2 + y^2 + z^2}) = (xdx + ydy + zdz)/\sqrt{x^2 + y^2 + z^2}$. For $x = 200, dx = 1, y = 200, dy = 2, z = 100, dz = -1$ this gives a change in the length of the diagonal of about $500/300 = 1.66\ldots$, so the new diagonal has length about 301.67.