Homework 5.

7.3 Radius of convergence is infinity by ration test, so "circle" of convergence is the whole plane.

7.12 Circle of radius 1, center 0 (use ratio test).

8.1 $e^{z_1}e^{z_2} = (1+z_1+z_1^2/2!+z_1^3/3!+\cdots)(1+z_2+z_2^2/2!+z_2^3/3!+\cdots) = 1+(z_1+z_2)+(\binom{2}{0}z_1^0z_2^2+\binom{2}{1}z_1^1z_2^1+\binom{2}{2}z_1^2z_2^0)/2!+(\binom{3}{0}z_1^0z_2^3+\binom{3}{1}z_1^1z_2^2+\binom{3}{2}z_1^2z_2^1+\binom{3}{3}z_1^3z_2^0)/3!+\cdots = 1+(z_1+z_2)^1/1!+(z_1+z_2)^2/2!+(z_1+z_2)^3/3!+\cdots = e^{z_1+z_2}$.

8.2 $\frac{d}{dz}e^z = \frac{d}{dz}(1+z+z^2/2!+z^3/3!+\cdots) = 1+2z/2!+3z^2/3!+\cdots = 1+z/1!+z^2/2!+\cdots = e^z$.

9.2 $e^{i\pi/2} = \cos(\pi/2)+i\sin(\pi/2) = i$

9.12 $4e^{-8\pi/3} = 4\cos(-8\pi/3)+i\sin(-8\pi/3) = 4(-1/2-i\sqrt{3}/2) = -2-2\sqrt{3}i$

9.19 $(1-i)$ has argument $-\pi/4$ and absolute value $\sqrt{2}$, so its 8'th power has argument -2π and absolute value $\sqrt{2}^8 = 16$. So $(1-i)^8 = 16$.

9.22 $(1-i)/\sqrt{2}$ has absolute value 1 and argument $-\pi/4$. So its 42nd power has absolute value $1^{42} = 1$, and argument $-42\pi/4 = -5(2\pi)-\pi/2$. So $((1-i)/\sqrt{2})^{42} = -i$.

9.27 $|e^{iy}| = \sqrt{e^{iy}e^{-iy}} = \sqrt{1} = 1$. So $|e^{x+iy}| = |e^x||e^{iy}| = |e^x|$.

9.28 $|re^{i\theta} = r$. So $|r_1e^{i\theta_1}r_2e^{i\theta_2}| = |r_1r_2e^{i(\theta_1+\theta_2)}| = r_1r_2 = |r_1e^{i\theta_1}||r_2e^{i\theta_2}|$. Proof for quotient is similar.

10.2 This is equal to $3\sqrt[3]{1}$, whose values are 3, $3(-1/2+\sqrt{3}i/2)$ and $3(-1/2-\sqrt{3}i/2)$. The points form an equilateral triangle in the complex plane.

10.18 $i = e^{i\pi/2}$ so its square roots are $e^{i\pi/2+2n\pi}$ which are $e^{i\pi/4} = (1+i)/\sqrt{2}$ and $e^{5i\pi/4} = -(1+i)/\sqrt{2}$.

10.22 By drawing $i-1$ we see that it has argument $3\pi/4$, so one cube root has argument $\pi/4$ and is therefore $\sqrt[3]{2}(1+i)/\sqrt{2}$. The other two roots are obtained by multiplying this root by the other 2 cube roots of 1.

10.28 $\cos(3\theta)+i\sin(3\theta) = e^{3i\theta} = (\cos(\theta)+i\sin(\theta))^3 = \cos(\theta)^3+3i\cos(\theta)^2\sin(\theta)-3\cos(\theta)\sin(\theta)^2-i\sin(\theta)^3$. Comparing real and imaginary parts we find $\cos(3\theta) = \cos(\theta)^3-3\cos(\theta)\sin(\theta)^2$ and $\sin(3\theta) = 3\cos(\theta)^2\sin(\theta)-\sin(\theta)^3$.

11.2 Adding the equations gives $\cos(\theta) = (e^{i\theta}+e^{-i\theta})/2$ and subtracting them gives $\sin(\theta) = (e^{i\theta}-e^{-i\theta})/2i$.

11.5 $e^{i\pi/4} = (1+i)/\sqrt{2}$. $e^{\log(2)/2} = 2^{1/2} = \sqrt{2}$. So $e^{i\pi/4+\log(2)/2} = 1+i$.

11.6 $\sin(i) = (e^{i^2}-e^{-i^2})/2i = (e^{-1}-e^1)/2i = i(e-1/e)/2$.

11.11 $\cos(2x)\cos(3x) = (e^{2ix}+e^{-2ix})(e^{3ix}+e^{-3ix})/4 = (e^{5ix}+e^{-5ix}+e^{ix}+e^{-ix})/4$ and each of the 4 terms in this sum has zero integral from $-\pi$ to π (using the fact that $e^{ia} = e^{ia+2n\pi i}$).

11.18 Done in class.

12.1 $\sin(x+iy) = \sin(x)\cos(iy)+\cos(x)\sin(iy) = \sin(x)\cosh(y)+i\cos(x)\sinh(y)$.