Homework 4.

4.1 Real part $= 1$, imaginary part$=1$, absolute value $= \sqrt{2}$, $\theta = \pi/4$ (plus any integral multiple of 2π). Complex conjugate $= 1 - i$.

4.7 Real part $= -1$, imaginary part$=0$, absolute value $=1$, $\theta = 0$ (plus any integral multiple of 2π). Complex conjugate $= -1$.

4.14 Real part $= \sqrt{2}$, imaginary part$=\sqrt{2}$, absolute value $=4$, $\theta = \pi/4$ (plus any integral multiple of 2π). Complex conjugate $= \sqrt{2} - \sqrt{2}i$.

5.1 Multiply numerator an denominator by $1-i$ to find that this number is $(1/2)-(1/2)i$. Real part $= 1/2$, imaginary part$=-1/2$, absolute value $= 1/\sqrt{2}$, $\theta = -\pi/4$ (plus any integral multiple of 2π). Complex conjugate $= 1/2 + i/2$.

5.6 $(1+i)/(1-i) = i$, so the square is -1. Real part $= -1$, imaginary part$=0$, absolute value $=1$, $\theta = 0$ (plus any integral multiple of 2π). Complex conjugate $= -1$.

5.13 Real part $= 5\cos(2\pi/5)$, imaginary part$=5\sin(2\pi/5)$, absolute value $= 5$, $\theta = 2\pi/5$ (plus any integral multiple of 2π). Complex conjugate $= 5\cos(2\pi/5) - 5\sin(2\pi/5)i$. (These expressions can be simplified slightly using $\cos(2\pi/5) = (\sqrt{5}-1)/4$, $\sin(2\pi/5) = \sqrt{10+2\sqrt{5}}/4$).

5.20 $1/(2-3i)^2 = -5/169 + (12/169)i$, $1/(x+iy)^2 = (x^2-y^2)/(x^2+y^2)^2 - 2ixy/(x^2+y^2)^2$.

5.32 $\sqrt{2^2+3^2}^4 = 169$.

5.39 $x = y =$ anything.

5.45 Some algebra gives $2xy = 0$, so either x or y is 0.

5.51 A circle, center 0, radius 2.

5.53 A circle, center 1, radius 1.

5.55 Put $z = x+iy$; then $x+iy-(x-iy) = 5i$, so $y = 5/2$. This gives a line parallel to the real axis passing through the point $5/2i$.

5.56 The positive half of the imaginary axis.

5.60 A circle, center $1-i$, radius 2.

5.62 An ellipse with foci 1 and -1 passing through the points ± 4 and $\pm\sqrt{15}i$.

6.3 Converges by ratio test.

6.4 Diverges as terms do not tend to 0 (they all have absolute value 1).

6.10 Converges as this is a geometric series with ratio of absolute value less than 1.

6.12 Converges by ratio test (or notice that this is series for $\exp(3+2i)$, and the series for exp converges everywhere).