Homework 3.

When doing hand calculations of series is is normal to give only the first two or 3 non-zero terms. I have given an excessive number in the solutions below just in case someone got carried away.

13.2 Multiply two power series: $x + x^2 + (1/3)x^3 - (1/30)x^5$

13.6 Multiply $\exp(x)$ by the series $1 + x + x^2 + ...$ to get $1 + 2x + (5/2)x^2 + (8/3)x^3 + (65/24)x^4 + ...$

13.8 Work out series for $1/\cos(x)$ as in lecture: $1 + (1/2)x^2 + (5/24)x^4 + (61/720)x^6 + (277/8064)x^8 + ...$ (There is no really simple formula for the general term.)

13.13 Substitute x^2 into the series for $\sin(x)$ to get $x^2 - (1/6)x^6 + (1/120)x^{10} - (1/5040)x^{14} + \cdots$

13.14 Divide series for $\sin(x)$ by x, then replace x by $\sqrt{x}$ to get $1 - (1/6)x + (1/120)x^2 - (1/5040)x^3 + \cdots$

13.16 Substitute series for $\log(1 + x)$ into the series for $\sin(x)$. If you managed to get more than 2 non-zero terms of this you are doing pretty well. Result is $x - (1/2)x^2 + (1/6)x^3 - (1/12)x^5 + (1/8)x^6 - \cdots$

13.17 Integrate the series for $\cos(t^2)$ term by term to get $x - (1/10)x^5 + (1/216)x^9 - ...$

13.33 Use method 1 of the hint to get $(-1/2)x^2 - (1/12)x^4 - (1/45)x^6 - (17/2520)x^8 - \ldots$

13.37 $e^x = e^3e^{x-3} = e^3(1 + (x - 3) + (x - 3)^2/2! + (x - 3)^3/3! + \cdots)$

13.40 $\sqrt{x} = \sqrt{25 + (x - 25)} = 5\sqrt{1 + (x - 25)/25} = 1 + (1/50)(x - 25) - (1/5000)(x - 25)^2 + (1/250000)(x - 25)^3 - \cdots$, using the binomial theorem to expand $(1 + y)^{1/2}$.

14.5 The series is alternating in this region, so the error is at most the first term omitted which is $x^4/24$. For x at most $1/2$, this is at most $(1/2)^4/16 = 0 \cdot 002604... < \cdot 003$.

14.6 The error is at most $|x|^2/2+|x|^3/3+|x|^4/4\cdots$, which is at most $|x|^2/2+|x|^3/2+|x|^4/2\cdots = |x|^2/2(1-x)$, and for $|x| \leq \cdot 1$ this is at most $(\cdot 1)^2/2(1 - \cdot 1) = \cdot 0055555\cdots < \cdot .0056$.

14.9 Sum is $(1-1/2)+(1/2-1/3)+(1/3-1/4)+\cdots = 1$. Sum of first n terms is $1-1/(n+1)$, so remainder after n terms is $1/(n + 1)$. So with 200 terms error is 1/201 which gives 2 decimal places accuracy. On the other hand the 200'th term is $1/200 \times 201$ which is much smaller: about $\cdot 000025$. So the size of the first term omitted is far smaller than the error.

15.5 $\log(1+x^3)$ is about $x^3 - x^6/2$, so its 4'th derivative is about $-6 \times 5 \times 4 \times 3x^2/2$. At $x = \cdot 2$ this is about -7.2. (In fact just taking the first non-zero term does not give a very good answer. Taking more terms gives a better answer -6.885)

15.11 The power series for $\int_0^x \cos(x^2)dx$ is $x - x^5/10 + x^9/240 - \cdots$. Taking the first 3 terms and putting $x = 1$ gives an answer of about $1 - 1/10 + 1/240 = \cdot 90\cdots$.

15.16 $\tan(x) = x + x^3/3 + \cdots$, so limit is 1/3 (= coefficient of x^3 in $\tan(x) - x$).

15.18 $1/(\exp(x) - 1) = x^{-1} - 1/2 + (1/12)x + \cdots$, so the limit is 1/2.

16.3 Converges by the integral test. The proof given is wrong because not all terms on the left are greater than the corresponding terms on the right.

16.18 Integrate $1 - u^2 + u^4 = u^6 \cdots$ term by term to get $x - x^3/3 + x^5/5 - x^7/7 + \cdots$.

16.22 By problem 16.18, this is $\arctan(1) = \pi/4$