

**Homework 21.**

7.1  $f(x) = 1/2 + \sum_{n=-\infty, odd}^{\infty} e^{inx} i / \pi n$

7.12 Done in lectures.

7.13  $a_n = c_n + c_{-n}$ ,  $b_n = (c_n - c_{-n})/i$ ,  $c_n = a_n/2 + b_n/2i$ ,  $c_{-n} = a_n/2 - b_n/2i$  ( $n \geq 0$ ),  $c_0 = a_0/2$ .

8.11

(a)  $f(x) = \pi^2/3 + 4 \sum_{n=1}^{\infty} (-1)^n \cos(nx) / n^2$

(b)  $f(x) = 4\pi^2/3 + 2 \sum_{n=1}^{\infty} (-1)^n e^{inx} (1/n^2 + i\pi/n)$

9.1

(a)  $\cos(x) + i \sin(x)$

(b)  $x \sinh(x) + x \cosh(x)$ .

9.6 Odd.  $f(x) = (4/\pi)(\sin(\pi x/l) + \sin(3\pi x/l)/3 + \sin(5\pi x/l)/5 + \dots)$ .9.9 Even.  $f(x) = 1/12 - (1/\pi^2)(\cos(2\pi x)/1^2 - \cos(4\pi x)/2^2 + \cos(3\pi x)/3^2 - \dots)$ .11.5  $\pi^2/8$ 11.6  $\pi^4/90$ 11.10 Calculate the average value of  $f(x)g(x)$  using the fact that  $\cos(nx)$  ( $n \geq 0$ ) and  $\sin(nx)$  ( $n > 0$ ) are pairwise orthogonal.

12.8 (a) 1/2 (b) 1

12.14

(a)  $f(x) = 1/3 + \sum_{n=1}^{\infty} 4 \cos(n\pi x) / \pi^2 n^2$

(b)  $\pi^4/90$ .