Homework 11.

8.1 If $f'(a)=0$ then near a, $f(x)$ is about $f(a)+x^x\times f''(a)/2!$. So if $f''(a)>0$ we have a minimum, and if $f''(a)<0$ we have a maximum.

8.2 We can change variables so $a=b=0$. Then $f(x,y)-f(0,0)$ is about $x^2f_{xx}/2+xyf_{xy}+y^2f_{yy}/2$. Recall that a quadratic form $ax^2+bxy+cy^2$ is positive definite if $b^2<4ac$ and $a>0$. This quadratic form is positive definite if $f_{xy}^2<4(f_{xx}/2)(f_{yy}/2)$ and $f_{xx}>0$, in other words $f_{xx}f_{yy}>f_{xy}^2$ and $f_{xx}>0$. So in this case the function has a minimum at $(0,0)$. The proof for a maximum is similar.

8.5 $f_x=1-2x-y$, $f_y=1-x-y$, so stationary point is at $(x,y)=(0,-1)$. $f_{xx}=-2$, $f_{xy}=-1$, $f_{yy}=-1$, so $f_{xx}f_{yy}>f_{xy}^2$ the point is either a maximum or a minimum. As $f_{xx}<0$ the point is a maximum.

9.1 $l=s,\theta=\pi/3$ (so we get a regular hexagon).

9.2 $r:l:s=\sqrt{5}:(1+\sqrt{5}):3$.

9.3 Maximize abc subject to $a+2b+2c=84$. Taking partial derivatives of $abc+\lambda(a+2b+2c-84)$ we get $bc+\lambda=0$ and $ab+2\lambda=0$ and $ac+2\lambda=0$. So $a=2b=2c$. So $a=28,b=14,c=14$ and $abc=5488$.

9.4 $4/\sqrt{3}$ by $6/\sqrt{3}$ by $10/\sqrt{3}$.

9.5 Taking partial derivatives of $\lambda(2x+3y+z-11)+4x^2+y^2+z^2$ gives $2\lambda+8x=0$, $3\lambda+2y=0$, $\lambda+2z=0$, $2x+3y+z-11=0$. Solution is $(x,y,z)=(1/2,3,1)$.

9.6 $xyz-\lambda(2x+3y+4z-6)$ gives $yz=2\lambda$, $xz=3\lambda$, $xy=4\lambda$. Solution is $(x,y,z)=(1,2/3,1/2)$ giving the volume to be $1/3$.

9.7 $xyz-\lambda(ax+by+cz-d)$ gives $yz=a\lambda$, $xz=b\lambda$, $xy=c\lambda$. Solution is $(x,y,z)=(d/3a,d/3b,d/3c)$ giving the volume to be $d^3/27abc$.

9.8 $((x-1)^2+y^2)+((x+1)^2+y^2)+\lambda(2x+3y-4)$ gives $4x+2\lambda=0$, $4y+3\lambda=0$, $2x+3y-4=0$. So $(x,y)=(8/13,12/13)$.

10.1 1 (Draw the graph and look!)

10.2 $x^2+y^2-\lambda(5x^2-6xy+5y^2-32)$ gives $x=y=\pm\sqrt{8}$ or $x=-y=\pm\sqrt{2}$, so the largest and smallest distances are 4 and 2.

10.7 $z+\lambda(2x+4y-5)+\mu(x^2+z^2-2y)$ gives $2\lambda+2x\mu=0$, $4\lambda-2\mu=0$, $1+2\mu z=0$. Solution is $x=-1/2,y=3/2,z=\pm\sqrt{11/4}$. So largest z is $\sqrt{11/4}$.

10.10

(a) Max $1/2$, min $-1/2$.

(b) Max 1 , min $-1/2$.

(c) Max 1 , min $-1/2$.