1.2 Done in class.

1.5 $.583333 = 58/100 + 3 \times (1 + 1/10 + 1/100 + ...)/1000 = 7/12$. (Or note that $.583333 \times 3 = 1.74999999... = 7/4$.)

2.6 $1 + 1/2 + 1/6 + 1/20 + 1/70....$

2.7 $\sum_{n=1}^{\infty} 2^{n-1}/(2n+1)$.

4.1 Remainder $|S - S_n| \leq 1/2^n$ (in fact it is equal to $1/2^n$). So for example, choose $\epsilon = 1/1000000$. Then we can take $N = 30$ (say). (This is not the best possible value of N.)

4.4 For $n \geq 3$, remainder is smaller than for the geometric series in question 1 by the hint, so we can use the same ϵ, N as in 4.1.

4.7 Remainder $S - S_n$ is less than $1/(n-1)$ by integral test, so if we put (say) $\epsilon = 1/1000000$ we can take $N = 1000002$.

5.1 Divergent, as limit is not 0.

5.8 Limit of terms is 0, so preliminary test says nothing. (In fact series is divergent.)

6.1 $n! \geq 24 \times 5 \times 6 \times \cdots \times n > 16 \times 2 \times 2 \cdots \times 2 = 2^n$ if $n \geq 4$.

6.2 $1 + (1/2) + (1/3 + 1/4) + (1/5 + 1/6 + 1/7 + 1/8) + \cdots \geq 1 + (1/2) + (1/4 + 1/4) + (1/8 + 1/8 + 1/8 + 1/8) + \cdots = 1 + (1/2) + (1/2) + (1/2) + \cdots = +\infty$.

6.3 $1 + (1/2^2 + 1/3^2) + (1/4^2 + 1/5^2 + 1/6^2 + 1/7^2) + \cdots \leq 1 + (1/2^2 + 1/2^2) + (1/4^2 + 1/4^2 + 1/4^2 + 1/4^2) + \cdots = 1 + (2/2^2) + (4/4^2) + (8/8^2) + \cdots = 1 + 1/2 + 1/4 + 1/8 + \cdots = 2$.

6.5 (a) Diverges; the terms are larger than those of the divergent series $1 + 1/2 + 1/3....$ (b) Diverges; the terms are larger than those of the divergent series $1 + 1/2 + 1/3....$

6.7 Diverges; integral is $\log\log(n)$ which tends to infinity with n.

6.8 Diverges; integral is $\log(n^2 + 4)/2$ (substitute $y = n^2 + 4$).

6.11 Converges; integral is $-2(1 + log(n))^{-1/2}$ which is bounded as n tends to infinity.

6.12 Converges; integral is $-1/2(n^2 + 1)$.

6.15 Done in class; integral is $\log(n)$ if $p = 1$, $n^{1-p}/(1-p)$ if $p \neq 1$.

6.16 The integral should have lower limit 1, not 0. The integral diverges because it is bad at 0, which has nothing to do with the convergence of the sum.

6.17 $\int_1^{\infty} e^{-n^2} dn \leq \int_1^{\infty} e^{-n} dn$ because $n^2 \geq n$ for $n \geq 1$, and the latter integral converges (and has value e^{-1}). (By the way, the hint is slightly wrong: the integral $\int_0^{\infty} e^{-n^2} dn$ **can** be evaluated explicitly, and it has value $\sqrt{\pi}/2$. However this is a very hard integral to do, and it is much easier just to bound it from above which is all that is needed.)