

Quiz - Math 53  
November 12, 2009

Solutions

1) Let  $C$  be the path around the square with vertices  $(1, 1)$ ,  $(1, -1)$ ,  $(-1, -1)$  and  $(-1, 1)$ . Compute the integral

$$\int_C \frac{1}{x^2 + y^2} ds$$

*Note that this integral is symmetric for all four sides of the square. So we can compute it for one side and multiply by 4. We can parameterize the right side of the square by*

$$x(t) = 1 \quad y(t) = t \quad -1 \leq t \leq 1$$

*Then  $x'(t) = 0$  and  $y'(t) = 1$ . This gives the integral for the right side as*

$$\int_{-1}^1 \frac{1}{1+t^2} \sqrt{0^2+1^2} dt = \tan^{-1}(t) \Big|_{-1}^1 = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

*So the integral over the whole square is 4 times this, or  $2\pi$ .*