

Math 113 Homework 8  
due Monday August 10, 2009

- (1) Do exercise 15.24 in Judson
- (2) Prove that  $\mathbb{Z}[x]/\langle x^4 + 12 \rangle$  is an integral domain but not a field.
- (3) Do exercise 19.1 in Judson
- (4) Do exercise 19.5 in Judson
- (5) Do exercise 19.14 in Judson
- (6) An automorphism of a field  $F$  is an isomorphism from  $F$  to itself. Prove that if  $f$  is an automorphism of  $\mathbb{Q}(\sqrt{2})$ , then for all  $a, b \in \mathbb{Q}$ ,  $f(a + b\sqrt{2}) = a \pm b\sqrt{2}$ . Conclude that  $\mathbb{Q}(\sqrt{2})$  has exactly two automorphisms.
- (7) Do exercise 19.26 in Judson
- (8) Let  $F \subset E$  be fields, and  $a(x)$  and  $b(x)$  are polynomials over  $F$ . Prove that  $a(x)$  divides  $b(x)$  in  $E[x]$  if and only if  $a(x)$  divides  $b(x)$  in  $F[x]$ .