

Math 113 Homework 7
due Tuesday August 4, 2009

- (1) Find all the ideals of \mathbb{Z}_{12} . Which ideals are prime? Which are maximal?
- (2) Given two rings, R_1 and R_2 , we can define a direct product $R_1 \times R_2$ of order pairs (r_1, r_2) with $r_1 \in R_1$ and R_2 .
 - (a) How should we define addition and multiplication in $R_1 \times R_2$? What are 0 and 1 in $R_1 \times R_2$? Prove that your answer is actually a ring.
 - (b) Find an ideal in $\mathbb{Z} \times \mathbb{Z}$ which is not principle.
- (3) Do exercise 14.16 in Judson
- (4) Do exercise 14.35 in Judson
- (5) Do exercise 14.41 in Judson
- (6) Prove or disprove:
 - (a) The intersection of two ideals is always an ideal.
 - (b) The intersection of two prime ideals is always a prime ideal.
- (7) Do exercise 16.15 in Judson